The Relationship Between the Length of the Base Period and Population Forecast Errors

STANLEY K. SMITH and TERRY SINCICH*

The base period of a population forecast is the time period from which historical data are collected for the purpose of forecasting future population values. The length of the base period is one of the fundamental decisions made in preparing population forecasts, yet very few studies have investigated the effects of this decision on population forecast errors. In this article the relationship between the length of the base period and population forecast errors is analyzed, using three simple forecasting techniques and data from 1900 to 1980 for states in the United States. It is found that increasing the length of the base period up to 10 years improves forecast accuracy, but that further increases generally have little additional effect. The only exception to this finding is long-range forecasts of rapidly growing states, in which a longer base period substantially improves forecast accuracy for two of the forecasting techniques.

KEY WORDS: Characteristics of population projections; Forecast accuracy and bias; Population projections and forecasts.

1. INTRODUCTION

The base period of a population forecast may be defined as the time period from which historical data are collected and statistical relationships are formulated to provide forecasts of future population values. A recent discussion of population forecast errors raised the question of how changes in the length of the base period might affect the accuracy and bias of population forecasts (Beaumont and Isserman 1987; Smith 1987). Although many studies have investigated the effects of differences in forecasting models, sources of data, size of place, and length of forecast horizon on population forecast errors, few have considered the potential impact of changes in the length of the base period. This study analyzes the relationship between the length of the base period and population forecast errors, using three simple forecasting techniques and data from 1900 to 1980 for states in the United States.

In this article, a population forecast is defined as the future population value produced by a particular forecasting technique and set of base data, and forecast error refers to the percentage difference between a forecast and the actual population for the same year. The following terminology is used to describe population forecasts.

1. Base year: the year of the earliest observed population size used to make a forecast.
2. Launch year: the year of the latest observed population size used to make a forecast.
3. Target year: the year for which population size is forecasted.
4. Base period: the interval between base year and launch year.
5. Forecast horizon: the interval between launch year and target year.

* Stanley K. Smith is Associate Professor of Economics and Associate Director, Bureau of Economic and Business Research, University of Florida, Gainesville, FL 32611. Terry Sincich is Assistant Professor of Information Systems and Decision Sciences, College of Business Administration, University of South Florida, Tampa, FL 33620. An earlier version of this article was presented at the annual meeting of the Southern Demographic Association, San Antonio, 1988.

2. DATA AND TECHNIQUES

The data used in this study were taken from U.S. Census Bureau reports showing decennial census counts and annual intercensal estimates for states in the United States from 1900 to 1980 (U.S. Bureau of the Census 1956, 1965, 1971, 1976, 1982, 1984). These reports covered all states and the District of Columbia from 1950 onward, and all except Alaska and Hawaii from 1900 to 1949. The data refer to total population only; no analysis was performed on age, sex, race, or other characteristics of the population.

The intercensal estimates made by the Census Bureau were based on statistical series that reflect changes in population size. For all decades, estimates were based on annual data on births, deaths, and school enrollment. For some decades, five-year migration data from the decennial census were used as well. In a few instances, data from special censuses were included. In recent years, annual data from federal income-tax returns and Medicare records were included. All intercensal estimates were controlled to ensure that they were consistent with decennial census counts. Although they certainly contain some errors (especially for years prior to 1930), we believe these estimates are quite reliable and that they provide a useful basis for investigating the effects of the length of the base period on population forecast errors.

Three simple extrapolation techniques were used to produce population forecasts. First was linear extrapolation (LINE), which assumes that a population will increase (decrease) by the same number of persons in each future year as the average annual increase (decrease) during the base period:

$$\hat{P}_t = P_o + x/y(P_o - P_b), \qquad (1)$$

where $\hat{P}_t$ is the state population forecast for the target year, P_o is the state population in the launch year, P_b is the state population in the base year, x is the number of years in the forecast horizon, and y is the number of years in the base period.

Journal of the American Statistical Association
June 1990, Vol. 85, No. 410, Applications and Case Studies

The second technique was exponential extrapolation (EXPO), which assumes that a population will increase (decrease) at the same annual percentage rate in each future year as it increased (decreased) during the base period:

$$\hat{P}_t = P_o \exp(rx), \quad (2)$$

where r is the average annual growth rate during the base period.

In the third technique (SHIFT), state population data were expressed as shares of the national population. These shares were obtained from historical data and extrapolated into the future by assuming that the average annual absolute change in a state's share of national population observed during the base period will continue throughout the forecast horizon. The extrapolated state shares were then applied to an independent forecast of national population to provide state population forecasts:

$$\hat{P}_t = \hat{P}_{jt}[P_o/P_{jo} + x/y(P_o/P_{jo} - P_b/P_{jb})], \quad (3)$$

where $\hat{P}_{jt}$ is the national population forecast for the target year, P_{jo} is the national population in the launch year, and P_{jb} is the national population in the base year.

The SHIFT technique requires an independent forecast of the population of the United States. Although researchers have been making national population projections for many years (e.g., Pritchett 1891; Whelpton 1928), methodologies have changed considerably over time. For this study we wanted a set of national forecasts based on the same methodology for all decades. We created such a set by applying the LINE and EXPO techniques to the U.S. population and taking the average as a national forecast. This is a very simple approach to population forecasting, but it is adequate for our purposes because SHIFT forecasts for states are not particularly sensitive to the choice of a national forecast.

Although simple techniques are frequently used for small-area population forecasts, they are no longer commonly used for state forecasts, having been replaced by more sophisticated cohort–component and economic–demographic techniques. There is no evidence, however, that more sophisticated techniques produce more accurate forecasts of total population than simple techniques (e.g., Ascher 1978; Kale, Voss, Palit, and Krebs 1981; Makridakis 1986; Murdock, Leistritz, Hamm, Hwang, and Parpia 1984; Smith 1984). Furthermore, these and other simple techniques have been used in numerous studies of population forecast accuracy (e.g., Greenberg 1972; Isserman 1977; Schmitt and Crosetti 1951, 1953; Smith 1987; Smith and Sincich 1988; Voss and Kale 1985; White 1954). Simple techniques are particularly suitable for the present study because their small data requirements allow them to be applied retroactively to all states for all time periods, providing comparable sets of forecasts based on identical assumptions. We believe these techniques provide a useful basis for investigating the relationship between the length of the base period and population forecast errors.

3. EMPIRICAL ANALYSIS

Using these techniques and population data from 1900 to 1980, forecasts were made for 10-, 20-, and 30-year time horizons, using base periods of 1, 5, 10, 20, 30, and 40 years. Thus there were 18 possible combinations of the base period and the forecast horizon. The number of forecasts in each combination was limited by using only years ending in 0 or 5 since 1910 as launch years. Table 1 shows the number of observations (i.e., state forecasts) included in each combination of the base period and forecast horizon, for the entire sample and for states divided by population size in the launch year and rate of population growth during the decade immediately preceding the launch year.

Forecast error (F_t) was calculated as the percentage difference between the population forecast ($\hat{P}_t$) and the true population (P_t) in the target year:

$$F_t = [(\hat{P}_t - P_t)/P_t]\, 100. \quad (4)$$

Table 1. Number of State Forecasts, by Length of the Base Period and Forecast Horizon, for States Divided According to Population Size and Growth Rate

Size/growth categories	Length of forecast horizon	Length of base period: 1	5	10	20	30	40
All states	10	645	645	643	541	441	343
	20	543	543	541	441	343	245
	30	441	441	441	343	245	147
Large/slow	10	369	369	369	321	264	203
	20	307	307	307	259	202	141
	30	258	258	258	210	153	92
Small/slow	10	145	145	143	131	105	76
	20	122	122	120	109	84	55
	30	102	102	102	91	66	37
Large/rapid	10	68	68	68	54	47	41
	20	57	57	57	43	36	30
	30	37	37	37	23	16	10
Small/rapid	10	63	63	63	35	25	23
	20	57	57	57	30	21	19
	30	44	44	44	19	10	8

NOTE: Large indicates a population of 1 million or greater. Small indicates a population of less than 1 million. Rapid denotes a growth rate of 25% or greater. Slow denotes a growth rate of less than 25%.

Two measures of forecast accuracy and bias were used. Mean absolute percentage error (MAPE) is the average percentage error when the direction of the error is ignored; this provides a measure of accuracy. Mean algebraic percentage error (MALPE) is the average percentage error when the direction of error is accounted for; this provides a measure of bias. A positive error indicates a tendency for forecasts to be too high and a negative error indicates a tendency for forecasts to be too low. We assumed that true state population numbers were those published by the U.S. Bureau of the Census; that is, no attempts were made to adjust for estimation or enumeration error.

Absolute Percentage Error. Figure 1 shows the relationship between MAPE and the length of the base period for each technique and forecast horizon. For LINE and SHIFT, the MAPE declined as the base period increased from 1 to 5 years and from 5 to 10 years, but remained very stable thereafter. This pattern was found for all three forecast horizons, but was most notable for the 20- and 30-year horizons. For EXPO, the same pattern was found for the 10-year horizon, but for the 20- and 30-year horizons the MAPE continued to decline as the base period increased to 40 years. The declines were very small for increases beyond 20 years, however.

In addition, we analyzed the relationship between MAPE and the length of the base period for short-run forecast horizons of one and five years (not shown here). For all three techniques, MAPE's for one-year horizons were virtually identical for base periods of 1, 5, 10, and 20 years; they were slightly larger for base periods of 30 and 40 years. For five-year horizons, there was a small but clear U-shaped relationship between MAPE and the length of the base period: For all three forecasting techniques, forecasts derived from 10-year base periods had smaller MAPE's than forecasts derived from either shorter or longer base periods. For both the one- and five-year horizons, however, differences in MAPE's by length of base period were very small. It appears that for short forecast horizons, the length of the base period has very little effect on mean forecast accuracy. In the remainder of this article we focus solely on the longer forecast horizons of 10, 20, and 30 years.

Figure 1 shows some large differences in MAPE's among forecasts with varying lengths of the base period. Are these differences statistically significant? To answer this question, we conducted formal statistical tests for each technique and forecast horizon. Two null hypotheses were considered. The first ($H1$) was that MAPE's were identical for forecasts derived from all six base periods:

$$H1\colon \Theta_1 = \Theta_5 = \Theta_{10} = \Theta_{20} = \Theta_{30} = \Theta_{40}, \tag{5}$$

where Θ_y is the mean error for the base period of length y years. Forecasts from one- and five-year base periods were then excluded, and the second hypothesis ($H2$) was that MAPE's were identical for forecasts derived from 10-, 20-, 30-, and 40-year base periods:

$$H2\colon \Theta_{10} = \Theta_{20} = \Theta_{30} = \Theta_{40}. \tag{6}$$

Using a data set very similar to that used in this study, Smith and Sincich (1988) showed that the distribution of algebraic forecast errors tended to be normal, but that absolute forecast errors had skewed (nonnormal) distributions, truncated at 0. Consequently, the normality assumption required by traditional one-way analysis of variance F tests for MAPE is likely to be violated. We therefore tested $H1$ and $H2$ using a nonparametric alternative, the Kruskal–Wallis H test.

The results of these tests are summarized in the left portion of Table 2. $H1$ must be rejected (at a significance level of .01) for all combinations of technique and forecast horizon, providing evidence that at least two of the MAPE's differed significantly from each other. $H2$, how-

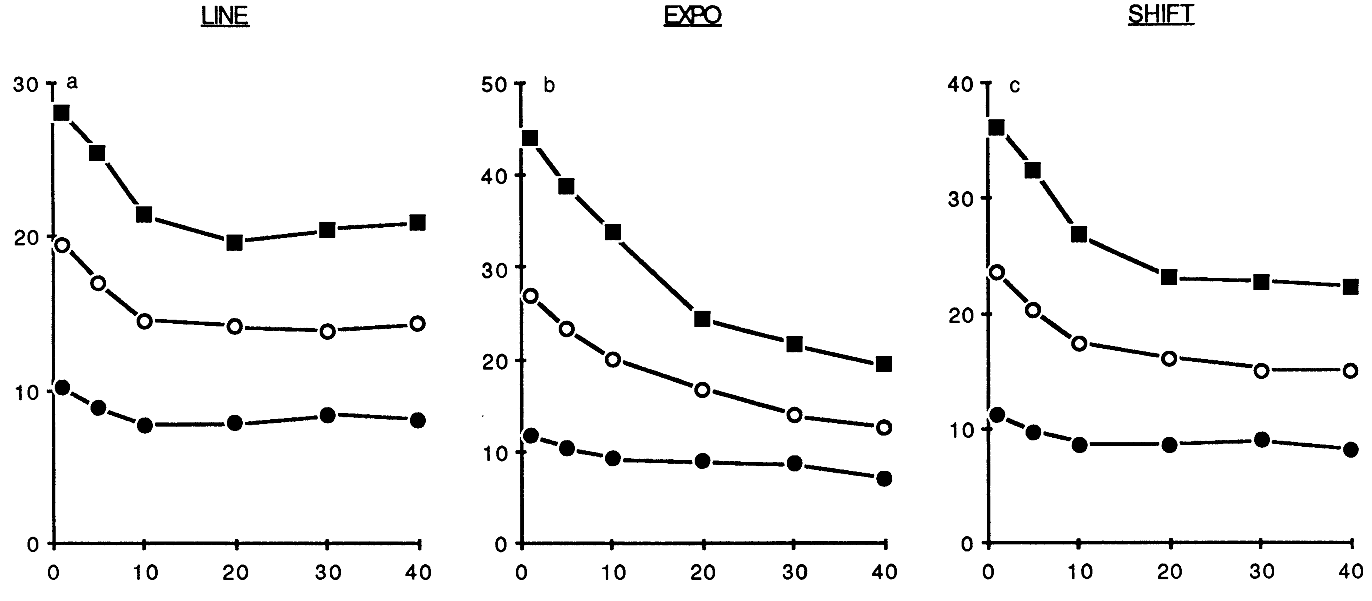


Figure 1. Mean Absolute Percentage Errors by the Length of the Base Period: (a) LINE; (b) EXPO; (c) SHIFT. The horizontal axis indicates the length of the base period and the vertical axis indicates MAPE. ●—● denotes 10-year forecasts, O—O denotes 20-year forecasts, and ■—■ denotes 30-year forecasts.

Table 2. *Tests of Significance for Differences in MAPE's, by Technique and Length of the Forecast Horizon*

		Nonparametric tests		Piecewise linear regression		
Technique	Horizon	H1	H2	Adjusted R^2	H3	95% CI(β_1 + β_2)
LINE	10	**	—	.91	—	—
	20	**	—	.98	—	—
	30	**	—	.94	—	—
EXPO	10	**	**	.92	*	−.126, −.009
	20	**	**	.98	**	−.350, −.155
	30	**	**	.95	*	−.774, −.147
SHIFT	10	**	—	.91	—	—
	20	**	—	.98	**	−.143, −.020
	30	**	—	.96	—	—

NOTE: For *H*1, MAPE's for 1-, 5-, 10-, 20-, 30-, and 40-year base periods are equal. For *H*2, MAPE's for 10-, 20-, 30-, and 40-year base periods are equal. For *H*3, the MAPE-base period slope over the interval (10, 40) is 0. * indicates that a hypothesis must be rejected at a significance level of .05. ** indicates that a hypothesis must be rejected at a significance level of .01.

ever, cannot be rejected for any forecast horizon for LINE and SHIFT. Only for EXPO must *H*2 be rejected. This suggests that for LINE and SHIFT, differences in the length of the base period had no significant effect on MAPE's, once the base period was 10 years or longer; whereas for EXPO, differences beyond 10 years did have a significant effect. As shown in Figure 1, however, the differences for EXPO-10 were very small and had little practical significance. (EXPO-10 is the exponential forecast with a 10-year horizon.)

The relationship between MAPE and the length of the base period can be investigated further by fitting a piecewise regression line through the six MAPE's for each technique and forecast horizon shown in Figure 1, with the slope changing when the base period reaches 10 years. Unlike the nonparametric tests of *H*1 and *H*2, which account for the variation of state forecast errors around the mean, the regression approach focuses solely on the means themselves. The piecewise linear regression model takes the form

$$\text{MAPE} = \beta_o + \beta_1 y + \beta_2(y - 10)I + \varepsilon, \qquad (7)$$

where y is the number of years in the base period, the indicator variable I takes the value 0 when $y \leq 10$ and the value 1 when $y > 10$, and ε is distributed normally with mean 0 and variance σ^2. Note that this model proposes different (but continuous) straight-line relationships over the two base-period intervals (1, 10) and (10, 40), with slopes represented by β_1 and $(\beta_1 + \beta_2)$, respectively.

In addition, if the length of the base period has no effect on MAPE's over the interval of 10–40 years, then the slope over this interval equals 0. We therefore test a third hypothesis:

$$H3\colon \beta_1 + \beta_2 = 0. \qquad (8)$$

The results of the piecewise regressions are shown in the right portion of Table 2. Tests of overall model adequacy are significant for each technique–horizon combination, and all R^2 values (adjusted for sample size) exceed .90. More important, *H*3 cannot be rejected (at a significance level of .05) for any horizon for LINE or for two of the three horizons for SHIFT. Only for EXPO and SHIFT-20 is there sufficient evidence to indicate that the slope of the line over the base-period interval (10, 40) differs from 0.

For each case in which *H*3 must be rejected, we report the associated 95% confidence interval for the slope, $\beta_1 + \beta_2$. All of the confidence intervals contain small negative numbers, indicating that the MAPE declined very slowly as the base period became longer. This was especially true for EXPO-10 and SHIFT-20. For EXPO-10, we expect (with 95% confidence) the MAPE to decline between .09% and 1.26% for every 10-year increase in the length of the base period; for SHIFT-20, we expect the decline to be between .20% and 1.43%. Even though differences in MAPE were sometimes statistically significant for base periods of 10 years or longer, we conclude that these differences generally had little practical significance.

To summarize the results, we note that Figure 1 and Table 2 show clearly that lengthening the base period from 1 to 5 years and from 5 to 10 years led to greater forecast accuracy for all three techniques and all three forecast horizons; these differences were often large and statistically significant (especially for 20- and 30-year horizons). After 10 years, however, increases in the length of the base period generally had little or no effect on forecast accuracy, even for the longest forecast horizon. The only exceptions were the 20- and 30-year forecasts from the EXPO technique, for which MAPE's continued to decline as the base period increased beyond 10 years. Even for these forecasts, the declines were very small after the base period surpassed 20 years.

It should also be noted that the data used to construct one- and five-year base periods *always* included at least one intercensal year, whereas the data used for longer base periods included decennial census counts and intercensal estimates in approximately equal proportions. Since decennial census counts are presumably more accurate than intercensal estimates, is it possible that the previously discussed findings reflect differences in data quality rather than differences in the length of the base period? To investigate this possibility, we compared forecast errors from 10-year base periods using data solely from decennial census counts (1900, 1910, 1920, . . .) with errors from 10-year base periods using data solely from mid-decade estimates (1905, 1915, 1925, . . .). We found the errors from these two sets of estimates to be very similar; there was no evidence that forecasts based on decennial census counts were more accurate than those based on intercensal estimates. We believe that one- and five-year base periods produced larger forecast errors than longer base periods because they covered a shorter time interval, not because they used less reliable data.

Algebraic Percentage Error. The analysis thus far has dealt with forecast accuracy, or errors regardless of sign. What about bias? Are there any patterns relating the length of the base period to the tendency for forecasts to be too high or too low? Figure 2 shows the relationship

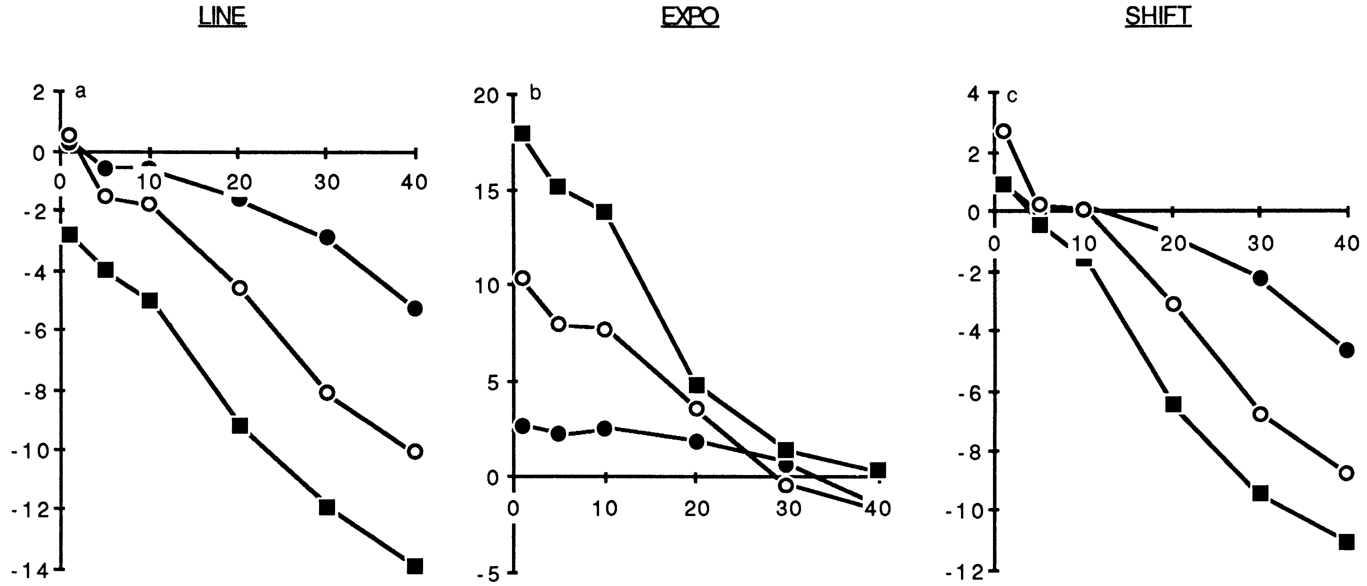


Figure 2. Mean Algebraic Percentage Errors by the Length of the Base Period: (a) LINE; (b) EXPO; (c) SHIFT. The horizontal axis indicates the length of the base period and the vertical axis indicates MALPE. ●—● denotes 10-year forecasts, ○—○ denotes 20-year forecasts, and ■—■ denotes 30-year forecasts.

between MALPE and the length of the base period. A strong negative relationship can be seen for all techniques and forecast horizons. What caused this negative relationship? We believe this may have been a spurious relationship caused by the very slow growth rates of the 1930s. Both absolute and percentage increases in the U.S. population were much smaller during the 1930s than any other decade of this century. The longer the base period, the larger the proportion of forecasts in the sample that included the 1930s as part of the base period. It is possible, then, that the negative relationship between MALPE and the length of the base period shown in Figure 2 simply reflected the more frequent inclusion of the 1930s in the base periods of forecasts with longer base periods.

To investigate this possibility, we deleted from the sample all forecasts that included years between 1930 and 1940 as part of the base period. The deletion of these forecasts had little effect on the results for MAPE but a large effect on the results for MALPE (Fig. 3). In contrast to Figure 2, all three techniques exhibited an upward bias; this bias was particularly strong for EXPO-20 and EXPO-30. More important, however, is the finding that there was no consistent relationship between MALPE and the length of the base period. In some instances MALPE's rose with an increase in the base period; in other instances they declined. Statistical tests showed that for MALPE's, $H1$ cannot be rejected for any technique or forecast horizon except EXPO-10 and EXPO-20, whereas $H2$ and $H3$ can-

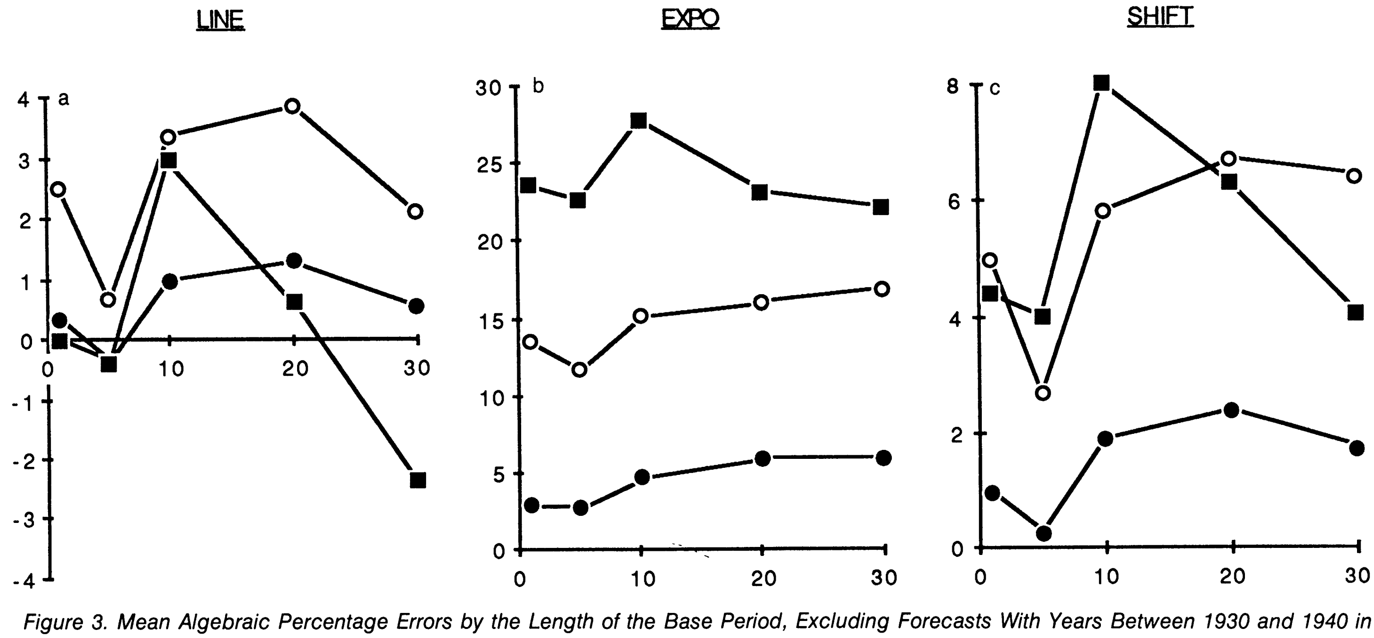


Figure 3. Mean Algebraic Percentage Errors by the Length of the Base Period, Excluding Forecasts With Years Between 1930 and 1940 in the Base Period: (a) LINE; (b) EXPO; (c) SHIFT. The horizontal axis indicates the length of the base period and the vertical axis indicates MALPE. ●—● denotes 10-year forecasts, ○—○ denotes 20-year forecasts, and ■—■ denotes 30-year forecasts.

not be rejected for any technique or forecast horizon at all.

We believe the strong negative relationship shown in Figure 2 between MALPE and the length of the base period was spurious, having been caused by the effects of the 1930s on population forecast errors. This study provides no evidence that the length of the base period has any particular effect on the degree of bias for population forecasts.

Analysis by Size and Growth Rate. Many studies have found that population size and growth rate have large effects on forecast accuracy and bias (e.g., Isserman 1977; Schmitt and Crosetti 1951; Smith 1987; White 1954). It is therefore possible that some of the results discussed before are not representative of all states, but only states with certain size or growth-rate characteristics. To test for this possibility, we separated states according to population size in the launch year (<1 million, ≥1 million) and growth rate during the decade immediately preceding the launch year (<25%, ≥25%). Forecast errors were then evaluated separately for each of the four resulting size/growth categories. Figure 4 shows the results for MAPE. As found in previous studies, large states generally had smaller errors than small states, and slowly growing states generally had smaller errors than rapidly growing states. These differences were occasionally quite large, especially for EXPO.

In general, the error patterns in Figure 4 are similar to those in Figure 1. For all techniques and size/growth categories, MAPE's declined as the base period increased from 1 to 5 years; for states with growth rates of less than 25%, they declined further as the base period increased from 5 to 10 years. For states with growth rates of 25% or more, however, lengthening the base period from 5 to 10 years frequently caused MAPE's to increase slightly. The effect of increases in the base period beyond 10 years varied by technique and size/growth category. For the LINE technique in all categories and the EXPO and SHIFT techniques in states with growth rates of less than 25%, lengthening the base period beyond 10 years either raised MAPE's or left them essentially unchanged. For the EXPO and SHIFT techniques in states growing by 25% or more, however, lengthening the base period from 10 to 20 years lowered MAPE's considerably, especially for the 20- and 30-year forecast horizons. Further increases beyond 20 years had relatively little additional effect. An explanation for this finding is given in the next section.

Nonparametric statistical tests identical to those reported earlier were applied to the forecast errors shown in Figure 4. The results are summarized in Table 3. For LINE and SHIFT, $H1$ can be rejected only for large, slowly growing states. For EXPO, $H1$ can be rejected for large states but generally cannot be rejected for small states. Although the differences in errors shown in Figure 4 are often large, small sample sizes and/or large variances prevent $H1$ from being rejected in some of the size/growth categories. $H2$ cannot be rejected for any size/growth categories for LINE and SHIFT, but can be rejected for large states for EXPO. These results indicate that except for EXPO forecasts in large states (especially rapidly growing states), there is no evidence that a base period of longer than 10 years had any significant effect on MAPE's.

MALPE's were calculated for each size/growth category as well (not shown here). When all forecasts were included, the results were similar to those in Figure 2: There was a clear negative relationship between MALPE and the length of the base period for each size/growth category. When forecasts with years between 1930 and 1940 in the base period were deleted, however, the results were quite different: For rapidly growing states there was generally a negative relationship between MALPE and the length of the base period, whereas for slowly growing states there was generally a positive relationship. We believe this reflects the empirical finding that places with very high growth rates during one decade usually have lower rates the following decade, whereas places with very low growth rates during one decade usually have higher rates the following decade (Smith 1987).

For slowly growing states, differences in MALPE's by length of the base period were typically quite small. For rapidly growing states they were occasionally quite large, especially for the 30-year forecast horizon and for the EXPO and SHIFT techniques. In most instances, however, the distribution of errors around the mean was very wide. Using the nonparametric tests described earlier, we found differences in MALPE's by length of the base period to be statistically insignificant in every instance for SHIFT, in every instance but one for LINE, and in most instances for EXPO. We conclude that the effects of the length of the base period on the degree of bias are generally small and/or statistically insignificant, even when states are divided by size and growth rate.

Comparison With Other Studies. How do the results reported in this article compare with those found in other studies? Only a few studies have considered the relationship between the length of the base period and population forecast errors. White (1954) used a ratio technique similar to SHIFT for forecasts of states and found that a 60-year base period produced considerably larger errors than a 30-year base period. Smith (1984) used LINE, EXPO, and SHIFT techniques for forecasts of Florida counties and found that a 20-year base period produced larger errors and greater bias than a 10-year base period. Beaumont and Isserman (1987) made LINE and EXPO forecasts for states that grew by more than 20% during the decade preceding the launch year. For EXPO forecasts, they found that a 40-year base period produced smaller errors and less upward bias than a 10-year base period; for LINE forecasts, however, they found that a 40-year base period did not improve accuracy and led to considerably greater downward bias than a 10-year base period. Voss and Kale (1985) made forecasts of minor civil divisions in Wisconsin and found that a 30-year base period produced slightly more accurate forecasts than a 10-year base period for the EXPO technique. They also found that weighting more recent decades in the base period more heavily than more distant decades improved forecast accuracy.

The relationship between the length of the base period and population forecast error was not the major focus of any of these studies, and none performed in-depth analyses of this relationship. Furthermore, their results are not perfectly comparable with those reported in this study because of differences in assumptions, techniques, and testing procedures. Nevertheless, the results reported in these studies are consistent with those reported here: With the possible exception of the EXPO technique (especially in rapidly growing places), increasing the length of the base period beyond 10 years generally does not improve forecast accuracy or reduce bias.

4. CONCLUSIONS

In this study we found that increasing the base period from 1 to 5 years and from 5 to 10 years almost always improved forecast accuracy, often by large and statistically significant amounts. This effect was found for all three forecasting techniques and all three forecast horizons in the total sample of states and in most size/growth categories. Increases beyond 10 years, however, generally did not improve forecast accuracy. In fact, in several instances increases beyond 10 years caused MAPE's to increase

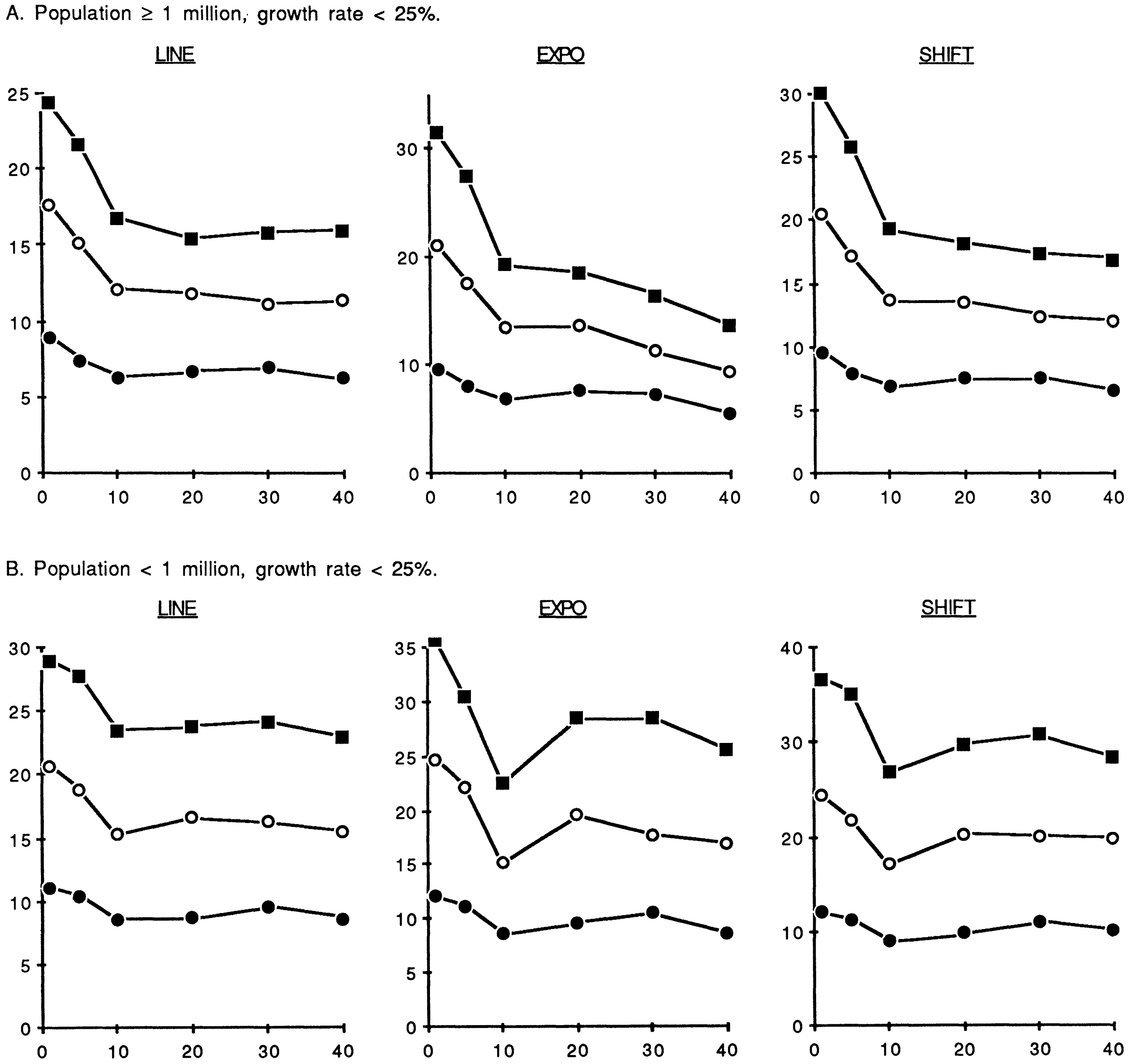
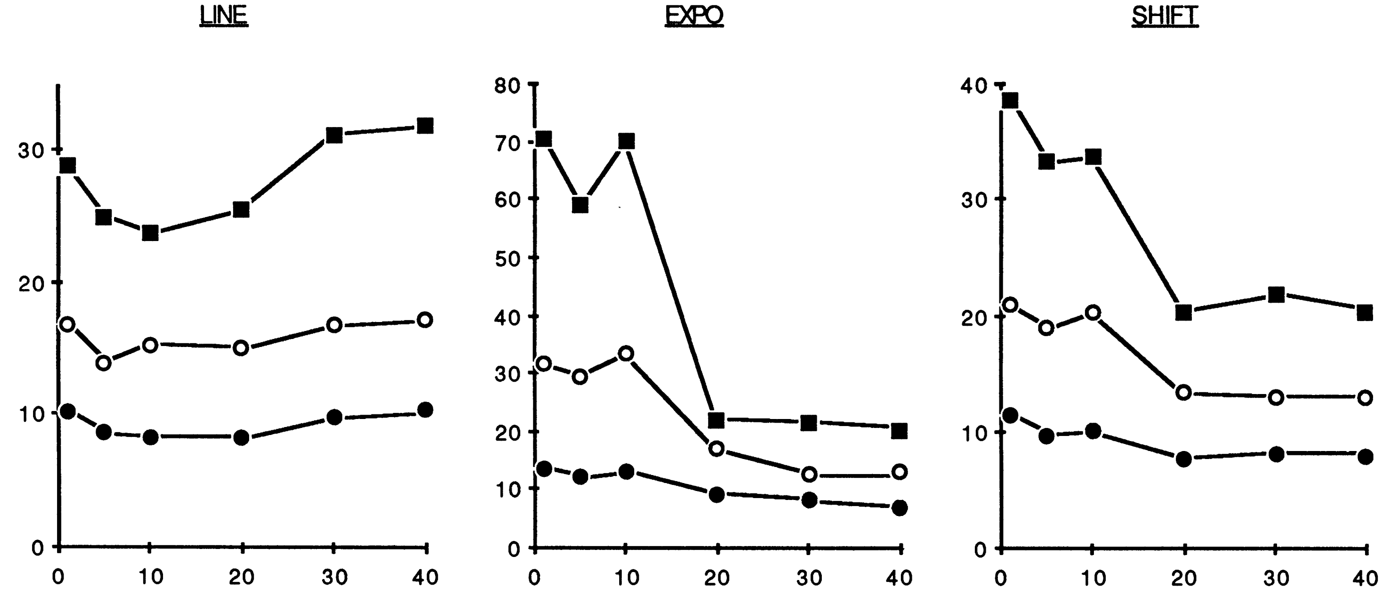


Figure 4. Mean Absolute Percentage Errors by the Length of the Base Period, for States Divided According to Population Size and Growth Rate: (a) Population Greater Than 1 Million, Growth Rate Less than 25%; (b) Population Less Than 1 Million, Growth Rate Less than 25%; (c) Population 1 Million or Greater, Growth Rate 25% or Greater; (d) Population Less Than 1 Million, Growth Rate 25% or Greater. The horizontal axis shows the length of the base period and the vertical axis shows MAPE. Population size refers to the launch year, and growth rate refers to the decade immediately preceding the launch year. ●—● denotes 10-year forecasts, O—O denotes 20-year forecasts, and ■—■ denotes 30-year forecasts.

C. Population ≥ 1 million, growth rate ≥ 25%.



D. Population < 1 million, growth rate ≥ 25%.

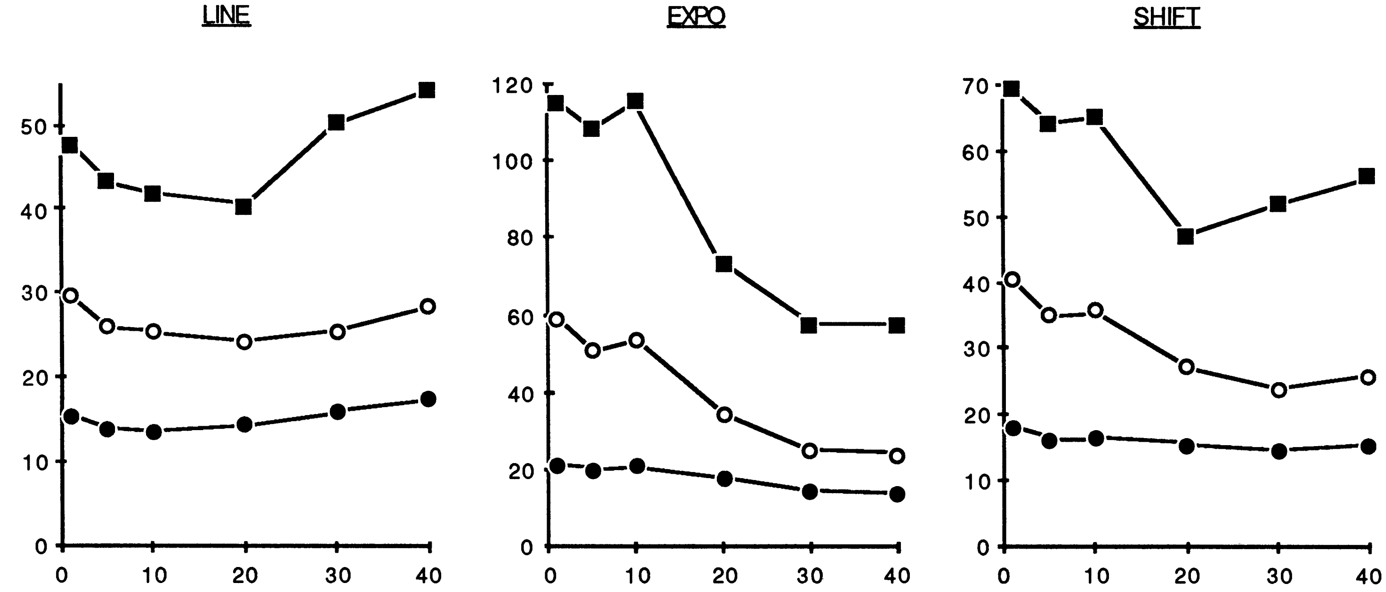


Figure 4 (continued)

rather than decline. We believe that too short a base period (e.g., one or five years) may incorrectly interpret short-run fluctuations as long-run trends, whereas too long a base period (e.g., 30 or 40 years) may reflect past statistical relationships that are no longer valid. We conclude that for simple extrapolation techniques, approximately 10 years of base data are generally sufficient to attain the highest possible degree of forecast accuracy.

The only exceptions to this conclusion were 20- and 30-year forecasts from the EXPO and SHIFT techniques in rapidly growing states. For these forecasts, increasing the base period from 10 to 20 years led to considerably smaller MAPE's, especially for 30-year forecast horizons. Furthermore, increasing the base period frequently led to substantial reductions in the upward bias of long-range forecasts for rapidly growing states. (These differences were not always statistically significant, perhaps because of small sample sizes for these size/growth categories.)

Why does a longer base period improve the forecasting performance of the EXPO and SHIFT techniques in rapidly growing states? We believe the explanation lies with the tendency for high growth rates to regress toward the mean over time. In an earlier study, Smith (1987) found that places with very high growth rates during one decade generally had lower rates the following decade. Since the EXPO technique forecasts a constant growth rate and the

Table 3. Nonparametric Tests of Significance for Differences in MAPE's, by Technique, Length of the Forecast Horizon, and Size/Growth Category

Technique	Length of forecast horizon	Size/growth categories: Large/slow	Small/slow	Large/fast	Small/fast
H1 (equal means, all base periods)					
LINE	10	**	—	—	—
	20	**	—	—	—
	30	**	—	—	—
EXPO	10	**	—	**	—
	20	**	—	**	**
	30	**	—	**	—
SHIFT	10	**	—	—	—
	20	**	—	—	—
	30	**	—	—	—
H2 (equal means, base periods 10, 20, 30, 40)					
LINE	10	—	—	—	—
	20	—	—	—	—
	30	—	—	—	—
EXPO	10	**	—	**	—
	20	**	—	**	**
	30	**	—	**	—
SHIFT	10	—	—	—	—
	20	—	—	—	—
	30	—	—	—	—

NOTE: Large indicates a population of 1 million or greater. Small indicates a population of less than 1 million. Rapid denotes a growth rate of 25% or greater. Slow denotes a growth rate of less than 25%. ** indicates that a hypothesis must be rejected at a significance level of .01.

SHIFT technique forecasts a growing state share of a growing national population (in rapidly growing states), both of these techniques frequently forecast too high when a short base period is used for states that are growing rapidly (especially for long forecast horizons). For states with high growth rates during a particular decade, then, using the EXPO and SHIFT techniques and one decade as a base period often leads to large errors (especially positive errors). Using a longer base period is one way to improve forecast accuracy and reduce bias for EXPO and SHIFT forecasts of rapidly growing populations. (Another solution to this problem is simply not to use these techniques for rapidly growing places.)

Can the conclusions drawn in this article be generalized to cover cohort–component, economic–demographic, and other population forecasting methods? Most forecasting methods rely on the extrapolation of trends in statistical relationships observed during the base period. We believe that when the base periods are the same, these extrapolated trends will generally be highly correlated with each other, regardless of the forecasting method employed. The results for LINE, EXPO, and SHIFT are illustrative: Although their functional forms differ, in most instances the relationships between forecast errors and the length of the base period were similar for all three techniques. Several other studies have found forecast error characteristics to be similar for different forecasting techniques as well (e.g., Kale et al. 1981; Murdock et al. 1984; Smith 1984; White 1954). Consequently, we believe that the relationship between forecast error and the length of the base period for other forecasting methods is likely to be similar to that reported here.

Further research on this topic is needed, not only covering other forecasting methods but other geographic areas as well. (Will the results for small areas differ from those reported here because of greater volatility in population growth rates?) We believe such research will improve our understanding of this important but largely neglected relationship and will provide valuable practical assistance to persons engaged in the production and interpretation of population forecasts.

[*Received November 1988. Revised November 1989.*]

REFERENCES

Ascher, W. (1978), *Forecasting: An Appraisal for Policy-Makers and Planners*, Baltimore: Johns Hopkins University Press.

Beaumont, P. M., and Isserman, A. M. (1987), Comment on "Tests of Forecast Accuracy and Bias for County Population Projections," by S. K. Smith, *Journal of the American Statistical Association*, 82, 1004–1009.

Greenberg, M. (1972), "A Test of Combinations of Models for Projecting the Population of Minor Civil Divisions," *Economic Geography*, 48, 179–188.

Isserman, A. (1977), "The Accuracy of Population Projections for Subcounty Areas," *Journal of the American Institute of Planners*, 43, 247–259.

Kale, B., Voss, P., Palit, C., and Krebs, H. (1981), "On the Question of Errors in Population Projections," unpublished paper presented at the annual meeting of the Population Association of America, Washington, D.C.

Makridakis, S. (1986), "The Art and Science of Forecasting: An Assessment and Future Directions," *International Journal of Forecasting*, 2, 15–39.

Murdock, S. H., Leistritz, F. L., Hamm, R. R., Hwang, S.-S., and Parpia, B. (1984), "An Assessment of the Accuracy of a Regional Economic–Demographic Projection Model," *Demography*, 21, 383–404.

Pritchett, H. S. (1891), "A Formula for Predicting the Population of the United States," *Quarterly Publication of the American Statistical Association*, 2, 278–286.

Schmitt, R., and Crosetti, A. (1951), "Accuracy of the Ratio Method for Forecasting City Population," *Land Economics*, 27, 346–348.

——— (1953), "Short-Cut Methods of Forecasting City Population," *Journal of Marketing*, 17, 417–424.

Smith, S. K. (1984), *Population Projections: What Do We Really Know?*, Monograph 1, University of Florida, Bureau of Economic & Business Research.

——— (1987), "Tests of Forecast Accuracy and Bias for County Population Projections" (with discussion), *Journal of the American Statistical Association*, 82, 991–1012.

Smith, S. K., and Sincich, T. (1988), "Stability Over Time in the Distribution of Population Forecast Errors," *Demography*, 25, 461–474.

U.S. Bureau of the Census (1956), "Estimates of the Population of States: 1900–1949," *Current Population Report*, Ser. P-25, No. 139.

——— (1965), "Revised Estimates of the Population of States and Components of Population Change 1950 to 1960," *Current Population Report*, Ser. P-25, No. 304.

——— (1971), "Preliminary Intercensal Estimates of States and Components of Population Change, 1960–1970," *Current Population Report*, Ser. P-25, No. 460.

——— (1976), "Estimates of the Population of States With Components of Change: 1970 to 1975," *Current Population Report*, Ser. P-25, No. 640.

——— (1982), "Provisional Estimates of the Population of States: July 1, 1981," *Current Population Report*, Ser. P-25, No. 911.

——— (1984), "Estimates of the Population of States: 1970 to 1983," *Current Population Report*, Ser. P-25, No. 957.

Voss, P. R., and Kale, B. D. (1985), "Refinements of Small-Area Population Projection Models: Results of a Test Based on 128 Wisconsin Communities," unpublished paper presented at the annual meeting of the Population Association of America, Boston.

Whelpton, P. K. (1928), "Population of the United States, 1925 to 1975," *American Journal of Sociology*, 34, 253–270.

White, H. R. (1954), "Empirical Study of the Accuracy of Selected Methods of Projecting State Population," *Journal of the American Statistical Association*, 49, 480–498.