ASSESSING THE ACCURACY OF TREND EXTRAPOLATION METHODS FOR POPULATION PROJECTIONS: THE LONG VIEW

Stefan Rayer

University of Florida

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ABSTRACT

This paper evaluates summary measures of population projection accuracy and bias for a large sample of counties and county equivalents in the continental United States over the period 1900–2000. The analysis has two primary purposes. The first is to investigate the relationship between accuracy and bias and the length of the projection horizon and base period. The second is to compare different trend extrapolation techniques with respect to their forecasting performance, both in the aggregate and by county size and growth rates. The study finds that the length of the base period has only a limited impact on accuracy and bias; that errors grow about linearly with increases in the projection horizon; that most projection methods provide comparable results for shorter projection horizons; that accuracy and bias vary by population size and growth rate; and that averages generally perform very well, equaling or exceeding the performance of individual techniques. The study confirms many of the findings of the earlier projection evaluation literature. By using a significantly enlarged dataset both with respect to space and time, its conclusions strengthen those previous studies and provide guidance regarding the production and interpretation of small area population projections.

INTRODUCTION

This study examines the accuracy and bias of trend extrapolation population projection techniques using one hundred years of county-level data. Despite the ascendance of cohort-component models, simple trend and ratio extrapolation methods remain popular, especially for small areas, where their ease of use, small data requirements, and reliability often compare favorably with more complex projection models. Various modeling options exist, and while some appear more appropriate than others when applied to areas following specific patterns of population change, the consensus seems to be that no particular method consistently provides the most accurate projection results. Many of the studies that have evaluated projection errors did so using a limited set of data both with regard to space and time (see e.g. Isserman 1977; Murdock et al. 1984; Smith 1987; Smith and Sincich 1991; Tayman et al. 1998). In particular, there exists a paucity of sub-state population projection evaluations that examine more than a few decades, and which are nationally representative. This is unfortunate, because small area population projections are used most frequently for actual planning purposes. The present study attempts to fill this gap by examining a wide range of popular trend extrapolation projection techniques using population data from every decennial census from 1900 to 2000 for all counties in the continental United States for which comparable data are available.

The analysis has two primary objectives. The first is to investigate the relationship between the length of the projection horizon and the base period with regard to accuracy and bias. Should the projection horizon and base period correspond in length or is a short base period, such as 10 years, generally sufficient? Using a rich dataset that covers the entire 20th century allows for an unprecedented number of projection horizon/base period combinations to be analyzed. The second objective is to compare the different trend extrapolation techniques with respect to their forecasting performance. In the first part of this section accuracy and bias are analyzed for all counties. Following that, the analysis further differentiates between counties representing different population size and growth groupings.

In addition to these specific aims, a more general objective of this paper is to understand how past trends of population change can inform projections of an unknown future. Simple trend extrapolation models are often not held in high regard among population forecasters. Yet numerous studies have found that more complex and sophisticated techniques are generally no more accurate (Long 1995; Murdock et al. 1984; Smith and Sincich 1992; Stoto 1983). While the past may not always repeat itself, this study argues that using historical data from a wide range of years, a broad sample of geographical units, and a rich cross-section of projection techniques aids both producers and users of small area population projections make better informed decisions.

DATA AND TECHNIQUES

This paper uses population data for counties and county equivalents in the continental United States – excluding Alaska and Hawaii – from the decennial U.S. censuses spanning the period 1900–2000. Throughout the 20th century many counties experienced changes to their boundaries that make a comparison of population figures from one census to the next problematic. In order to preserve comparability, the analysis was limited to those counties that did not experience significant boundary changes over the study period. To determine which of the changes were significant, this study follows Forstall, who identified the census date since which each "county has had no significant territorial change, that is, a boundary change large enough to have a significant effect on the county's population as of the preceding census" (1996:viii). This resulted in a total number of 2,482 counties that did not experience significant boundary changes between 1900 and 2000, which amounts to 79.0% of all the counties in Census 2000. To test whether this restricted sample is representative of the nation at large, the study compares results to those obtained with a larger sample of 2,978 counties for the sub-period 1930–2000, which represents 94.8% of all Census 2000 counties.

Covering census data for the entire 20^{th} century, the analysis involves 125 projection horizon/base period combinations spanning a range between 10 and 50 years. For each of these projection horizon/base period combinations, a total of 10 projection techniques were applied, including seven primary techniques and three averages. The primary techniques include linear (LIN), modified linear (MLN), share-of-growth (SHR), shift-share (SFT), exponential (EXP), constant-share (COS), and constant (CON). In addition, the study calculated three average projections comprising all seven trend extrapolation techniques (AV7), excluding the highest and the lowest projection (AV5), and excluding the two highest and two lowest projections (AV3). The seven primary techniques were calculated as follows:

LIN: In the linear extrapolation technique, it is assumed that the population will increase (decrease) by the same number of persons in each future decade as the average per decade increase (decrease) observed during the base period:

$$P_t = P_l + x / y\ (P_l - P_b),$$

Where P_t is the population in the target year, P_l is the population in the launch year, P_b is the population in the base year, x is the number of years in the projection horizon, and y is the number of years in the base period.

MLN: The modified linear extrapolation technique initially equals the linear method but in addition distributes the difference between the sum of the linear county projections and the independent national projection proportionally by population size at the launch year:

$$P_{it} = LIN + P_{il} / P_{jl} (P_{jt} – \Sigma LIN),$$

Where $_i$ represents the county and $_j$ the nation.

The modified linear method, as well as the share-of-growth, shift-share, and constant-share techniques, require an independent national projection for the target year population. Although population projections for the nation have been available for quite a long time (see e.g. Bonynge 1852; Pritchett 1891; Whelpton 1928), there exists no satisfactory set that covers all the target years used in this study. Instead, a new set was produced by applying the linear and exponential trend extrapolation techniques to the national population. To flatten out the discrepancies between the linear and exponential methods, an average of the two techniques was then calculated and used for the 4 ratio methods.

SHR: In the share-of-growth technique, it is assumed that the county's share of population growth will be the same over the projection horizon as it was during the base period:

$$P_{it} = P_{il} + [(P_{il} – P_{ib}) / (P_{jl} – P_{jb})] (P_{jt} – P_{jl})$$

SFT: In the shift-share technique, it is assumed that the average per decade change in each county's share of the national population observed during the base period will continue throughout the projection horizon:

$$P_{it} = P_{jt} [P_{il} / P_{jl} + (x / y) (P_{il} / P_{jl} – P_{ib} / P_{jb})]$$

EXP: In the exponential technique, it is assumed that the population will grow (decline) by the same rate in each future decade as it did, per decade, during the base period:

$$P_t = P_l\, e^{rx}, \qquad r = [\ln (P_l / P_b)] / y,$$

Where e is the base of the natural logarithm and ln is the natural logarithm.

COS: In the constant-share technique, it is assumed that the county's share of the national population will be the same in the target year as it was in the launch year:

$$P_{it} = (P_{il} / P_{jl})\, P_{jt}$$

CON: In the constant technique, it is assumed that the county population in the target year is the same as in the launch year:

$$P_t = P_l$$

SUMMARY MEASURES OF PROJECTION ACCURACY AND BIAS

This analysis focuses on the accuracy of the projections and on their bias. Both require choosing appropriate summary measures for their determination. According to the National Research Council (1980), any summary measure of error should meet the criteria of measurement validity, reliability, ease of interpretation, clarity of presentation, and support of statistical evaluation. With respect to accuracy, the most popular error measure in population forecasting is the Mean Absolute Percent Error or MAPE (see e.g. Ahlburg 1995; Isserman 1977; Smith 1987; Smith and Sincich 1988, 1990, 1992). The MAPE is popular, because it meets most of the above described desired criteria, although it has been criticized particularly with respect to its reliability and validity (Coleman and Swanson 2004; Swanson et al. 2000; Tayman and Swanson 1999).

Alternative measures of error that are sometimes used when evaluating population projections, and which all address some of the presumed shortcomings associated with the MAPE, include the Median APE, MAPE-R, and M-Estimators (Armstrong and Collopy 1992; Swanson et al. 2000; Campbell 2002; Tayman and Swanson 1999). Measures for bias are often computed analogously, e.g. the Mean Algebraic Percent Error (MALPE) alongside the MAPE, and the Median ALPE together with the Median APE.

In a companion study that used the same dataset, results for the MAPE, the Median APE, and an M-Estimator – as well as their counterparts for bias – provided generally comparable results (Rayer 2005). Of the three measures, the MAPE consistently reported the highest forecast errors, on average exceeding the Median APE by 30–40%, but overall the conclusions reached were similar. The two robust measures or error, for all intents and purposes, produced equivalent results. Whether the MAPE really overstates forecast error or whether the robust measures actually understate error is open to debate. After all, an inflated MAPE provides important information. However, for comparative purposes robust measures of forecast error are more useful. Because the analysis deals with a large sample, the issues with using a median-based measure of error are of less concern here, while its advantages – especially its familiarity and ease of interpretation – still apply. The analysis that follows will therefore be restricted to the Median APE and Median ALPE as summary measures of forecast accuracy and bias.

RESULTS

1900–2000 vs. 1930–2000

The population projections analyzed in this study based on decennial census data include every base year from 1900–1980, every launch year from 1910–1990, and every target year from

1920–2000, thus effectively covering the entire 20th century. As such, the study greatly extends earlier tests of forecast accuracy and bias by applying a wider range of dates than previously examined. While several studies have analyzed forecast errors of state projections going back to 1900 (see e.g. Smith and Sincich 1990, 1991), the range of years included at the county level has been much smaller (see e.g. Smith 1987). Going back to 1900 at the county level necessitated making choices in order to preserve the comparability of the data. In particular, it was decided to exclude all those counties from the analysis that experienced a significant boundary change at any point in time between 1900 and 2000. This resulted in a total N of 2,482, or about 79% of all counties in existence in the year 2000.

To check whether this sample remains representative of the universe of counties, the first part of the analysis examines results for the sub-period 1930–2000, for which comparable data are available for the majority of present day counties.[1] Table 1 displays Median APEs by projection horizon (10–50 years plus the overall average) for each of the 7 primary projection techniques plus the three averages. For each projection horizon, the top line represents results from the subset of counties that experienced no significant boundary change since 1900 (N=2,482), while the bottom line shows results for all counties that did not change after 1930 (N=2,978).

The table makes clear that the Median APEs between the two datasets are very similar. While for each projection horizon and each projection technique the Median APEs from the subset of counties that did not change since 1900 are somewhat smaller, in most instances the differences are not great. Moreover, for every projection horizon both datasets identify the same

[1] Starting the analysis in 1930 leads to a sample of 2,978 counties, or 94.8% of all counties in Census 2000. Since most of the boundary changes and the creation of new counties occurred before 1930, using a later starting date yields only small improvements in coverage, amounting to about 0.4% per decade between 1930 and 1950, and about 0.8% per decade thereafter. At the same time, the number of base period/projection horizons that can be analyzed decreases substantially with each later starting date.

projection technique providing the highest and the lowest MAPEs, thus leading to similar conclusions. The general comparability of the two samples is confirmed by an analysis of bias, using Median ALPEs as indicators (data not shown here).

Thus, although there are some small differences in overall projection accuracy whether one includes all the counties that have not experienced significant boundary changes since 1930, or only those that did not change since 1900, the conclusions reached are the same. In order to include the maximum number of projection horizons and base periods, the remainder of the paper will therefore focus on the subset of counties (N=2,482) that goes back to 1900. Starting in 1900 rather than 1930 more than doubles the number of projection horizons and base periods from 54 to 125, which is of great significance especially for the analysis of the longer projection horizons for which the 1930–2000 analysis would have provided very few cases or none at all.

Projection Horizon – Base Period Relationship

Few studies have specifically addressed the issue of the relationship between the length of the projection horizon and the length of the base period. A general recommendation is that the two should correspond to one another (Alho and Spencer 1997). Smith and Sincich (1990), in a study of 10–30 year projection horizons at the state level, found that while increasing the length of the base period up to 10 years improved forecast accuracy, further increases had little effect. The only exception were long-range forecasts (20–30 years) for rapidly growing states, where increasing the base period to 20 years resulted in considerably smaller MAPEs for projections using the exponential and shift-share techniques. Using a longer base period also reduced the upward bias of long-range forecasts for rapidly growing states. Beaumont and Isserman (1987) also found that lengthening the base period from one to four decades increased accuracy and reduced bias for state projections using the exponential method. It had the opposite effect on projections made with the linear method, which led them to the conclusion that "the choice of method and base period should be made together in a manner that compensates for the presence of regression to the mean" (1987:1006). This study revisits the relationship between the length of the base period and that of the forecast horizon with respect to both accuracy and bias. Because the data set covers the entire 20th century, significantly more base period/forecast horizon combinations are investigated at a lower level of geography than previously attempted.

Table 2 shows Median APEs by projection horizon and base period for the seven primary techniques plus the three averages. For each projection horizon the Median APEs are displayed for five different base periods, ranging from 10 to 50 years in length. Because CON holds the population constant at the launch year value there are no differences in Median APE among the five base periods.[2] For the other methods, the choice of the base period has some impact on forecast accuracy. The most consistent results are provided by COS, which exhibits an increase in forecast accuracy the longer the base period. For EXP, the ten year base period always produces the highest Median APE, with relatively little difference among the remaining longer base periods. Conversely, SFT performs worst with a 50 year base period, with little constancy among the other periods. The remaining three methods show no clear-cut pattern, except that the 10 year base period for all but the 20 year projection horizon have the highest errors. The three averaging methods do not show much variation, but the 10 year base period projections are associated with the largest Median APEs for the longest projection horizons as well.

Perhaps the most general impression that the data in Table 2 convey is how minor the differences in Median APE are among the five base periods. Except for COS, and to a lesser

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[2] COS holds the county's share of the national population constant at the launch year's value, but different base periods come into play in the form of the national projections used in their calculation.

extent SFT and EXP, the length of the base period seems to have a trivial impact on forecast accuracy. Smith and Sincich (1990) concluded that going beyond 10 years had a negligible impact on forecast accuracy. The results obtained in this study, executed at the county level and for more target years and longer projection horizons, lead to a similar conclusion, except that 20 years seems a better cut-off point, because extending the base period from 10 to 20 years yielded some improvement in accuracy for most models.[3]

Focusing on bias, Table 3 is structured analogously to Table 2, and shows the Median ALPEs by projection horizon, base period, and projection technique. Except for COS, all models show mostly negative coefficients, in other words the models tended to under-project population. The negative bias is most severe for SFT whereas EXP shows little bias. For most models, except COS, the Median ALPEs increase with lengthening base periods. This is in apparent contrast to Smith and Sincich (1990), who found no consistent relationship between MALPE and the length of the base period. However, it is believed that the pattern shown in Table 3 is the result of a spurious relationship. That is, it is not the length of the base period but rather the choice of launch and target years that determines bias.

To check for spuriousness, Median ALPEs were investigated by launch year for each projection horizon (data not shown). The general pattern was that launch year had a much greater impact on bias than the length of the projection horizon. For example, for a 10 year projection horizon all five projections with a 1990 launch year came out too low, all five projections with a 1980 launch year were too high, and all five projections with a 1970 launch year were too low.

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[3] Table 2 shows only a marginal improvement in terms of the Median APE for the exponential method when the base period is extended beyond 10 years. This hides the fact that going from 10 to 20 years has a significant impact for this method with respect to the potential for extreme outliers. For a 50 year projection horizon, extending the base period from 10 to 20 years reduced the MAPE from the millions to below 10,000, while going to 40 years brought the MAPE down to 612 (data not shown). While still inflated, and still representing highly unrealistic projections for individual counties, it demonstrates that extreme rates of population growth or decline tend to moderate over time, and that for counties with particular population change regimes the choice of the length of base period can make a significant difference.

Both the 1990s and especially the 1970s were periods of strong growth for the average county (see Table 4). In fact, the median county growth during the 1970s was the highest of any decade during the 20th century.[4] Thus, projections made for 1980 and 2000 that used extrapolations of population change of a previous decade consistently under-projected population for those target years. In contrast, projections for 1920–1960 in general turned out to be too high, which again can be explained by the higher average growth rates during the early years of the century shown in Table 4. The pattern is somewhat more complex to interpret for the longer projection horizons, but the general conclusion remains: the later target years, which saw rapid average growth rates over the projection horizon, had a strong tendency to be too low, which can be explained by the lower average growth rates experienced during the mid-century base periods upon which the projections were based.

The reason why the 10 and 20 year projections show less bias than the longer projection horizons is a function of the weight particular target years obtain in the calculation of the averages. For every projection horizon, there are more projections available for later target years than for earlier target years. For example, there are five 50 year projections for 2000 representing 10–50 year base periods, but only 1 for 1960. Because average growth rates were highest in the latter decades of the 20th century, most projections for these target years were too low. Yet the averages reported in Table 3, which show in general an increasing bias with increasing projection horizons and increasing base periods, are skewed towards these later years, because more projections are available for those years, thus creating this spurious relationship.

[4] This is different from population change for the nation overall, where the 1970s had the third lowest growth of any decade of the 20th century. Furthermore, while the 1940s, 1950s, and 1960s showed some of the highest growth rates for the nation, the average county – as represented by the median – actually grew very little during that period. Because each county in this study has the same weight with respect to the calculation of the accuracy and bias measures, it is the average county population change rather than the change for the nation that matters.

In reality it is very difficult if not impossible to establish the impact the length of the base period has on the bias for a given projection horizon. That is because bias for trend extrapolation methods is determined by both population changes over the projection horizon and the base period. If growth over the base period was stronger than between the launch and the target year, the projections will turn out too high, and vice versa. The same is not true with respect to accuracy. In general, accuracy of the projections decreases fairly linearly with increasing projection horizons. As seen in Table 2, for each ten year projection horizon the average absolute percent error increases by about 7–8%. Within each projection horizon, there is some variation due to different target years and different base period lengths, but compared to the bias measure the differences are minor, and they do not follow a specific pattern. Thus, while projection bias is largely determined by the differential population changes experienced over the base period and the projection horizon, the accuracy of the projections is most affected by the length of the projection horizon, with particular base, launch, and target years playing a less important role.

Accuracy and Bias by Trend Extrapolation Model

Tables 5a and 5b show the Median APEs and ALPEs for 10-50 year projection horizons with 20 year base periods by projection technique. The 20 year base period projections were chosen here because, as the data in Table 2 have demonstrated, there was a slight improvement in accuracy over the 10 year projections for most models. Going beyond 20 years made virtually no difference with respect to accuracy, and it also would have reduced the number of projection models left in the analysis.

Tables 5a and 5b demonstrate the value of averaging. All three averages produced projections with higher accuracy, as measured by the Median APE, than most primary techniques; indeed AV7 showed lower Median APEs than any primary technique for all projection horizons. The averages performed almost as well with respect to bias; only EXP and MLN were as good or better. Of the seven primary projection techniques, LIN, MLN, EXP, and CON showed the lowest Median APEs, and SFT and COS the highest. SFT and COS also had the most bias, which was strongly negative for SFT and strongly positive for COS.[5] EXP and MLN were the least biased of the individual techniques.

The data in Table 5a illustrate quite clearly how the accuracy of the projections declines with increasing projection horizons. This is a well established fact in the projections literature (see e.g. Keyfitz 1981; Smith and Sincich 1992; Stoto, 1983). For most projection methods the relationship between accuracy and the length of the projection horizon is linear or nearly linear (Smith and Sincich 1991). This is reflected in Table 5a, where for each of the ten techniques the Median APE goes up with increasing projection horizon. For all methods but SFT and COS, extending the projection horizon carries a penalty of about 6% to 8% for each ten years, while for the former the increase in error is higher.

In contrast to accuracy, with respect to bias there exists no consistent relationship to the length of the forecast horizon. According to Smith and Sincich (1991:272), "MALPEs differed from one forecasting technique to another, from one size-growth category to another, from one launch year to another, and over the length of the forecast horizon." Table 5b appears to prove otherwise – showing a generally increasing bias with lengthening forecast horizon – but the Median ALPEs of the individual projection horizon/base period combinations, upon which the averages shown in Table 5b are based, varied to such an extent that they are almost meaningless

[5] For most decades of the 20th century the population of the United States has become more concentrated. This can be seen by both rising Gini coefficients and by decreasing median county proportions of the national population (data not shown). COS holds each county's share of the national population constant at some past value, thus leading to projections that are too high on average.

(data not shown). Thus, while some projection techniques involve more bias than others, it is believed that there exists no consistent relationship to the length of the forecast horizon.

Accuracy and Bias by Trend Extrapolation Model, Population Size, and Population Growth

Following the general analysis of forecast accuracy and bias for all counties in the United States, the study now shifts to the investigation of county groupings that share similar population characteristics. The attributes most commonly investigated for this purpose include population size and the rate of population growth or decline. Both have been found to be important and consistent determinants of the accuracy of projections (Isserman 1977; Murdock et al. 1984; Smith 1987; Smith and Sincich 1988; Tayman et al. 1998; White 1954). As a general rule, projections made for larger places tend to be more accurate than those for smaller places, and projections made for slow to moderately growing places tend to be more accurate than those for fast growing and declining places. Up to now, these conclusions have been based on studies using a limited universe of counties and/or a limited study period. The present analysis extends previous studies by investigating the size/growth rate relationship to forecast accuracy and bias for all counties in the country that had comparable data available for the entire 20th century, thus greatly enhancing the ability to make generalizations about the findings.

In order to be most useful for practical purposes, population size in this study is measured at the launch year and the rate of population growth or decline refers to that over the base period. With respect to population size, the analysis distinguishes between six categories: less than 2,500, 2,500 to 7,500, 7,500 to 15,000, 15,000 to 30,000, 30,000 to 100,000, and more than 100,000 persons. For the rate of growth, ten categories are investigated: less than -15%, -15% to

-10%, -10% to -5%, -5% to 0%, 0% to 5%, 5% to 10%, 10% to 15%, 15% to 25%, 25% to 50%, and more than 50% growth per decade over the base period. In delineating these categories a compromise had to be made between having the finest possible gradation with respect to the population characteristics and having a sufficient number of cases in each category. The latter was important, because the analysis spans such a long time period, and the size and growth characteristics were applied uniformly to any launch year and base period.[6]

The data shown in Figures 1–4 focus on the LIN, SFT, EXP, CON, and AV7 methods, and only display results for 10 and 20 year projection horizons.[7] Overall, results for MLN and SHR were similar to LIN, while AV5 and AV3 were comparable to AV7. The results for COS were rather different from most other primary projection techniques, showing most affinity with the other constant technique, CON. However, because of the overall low accuracy and high bias of the method, as well as its rarity in actual use, it was not considered further in detail. Given that most sub-state population projections are produced for 20 year or shorter horizons, Figures 1–4 only display accuracy and bias for 10 and 20 year horizons.

Figure 1 displays the Median APE for the six population size categories. For each of the five methods, the bars are ordered from left to right representing increasing county population sizes. As expected, the accuracy of the county projections improves the larger the county. For both projection horizons and every technique except CON, the Median APE decreases consistently from the smallest to the second largest population size category, with the most significant increase in accuracy noted between counties having a population of less than 2,500

[6] The chosen delineations resulted in size categories that included at least 50 counties in every launch year, and in growth categories that included at least 30 counties. Most categories, however, were significantly larger. In a companion study (Rayer 2005), the same relationships were analyzed using three different summary measures of error and the analysis also included a visual examination of the individual APEs and ALPEs. The relatively small number of cases in some size/growth categories did not affect the results as to preclude the use of a median-based summary measure of error.
[7] Although not presented in Figures 1–4, the three other primary projection techniques remain in the analysis and were used, as before, in the calculation of the averages.

and those having a population of 2,500 to 7,500 persons. Interestingly, for every method and both projection horizons, the Median APEs increase again for counties in the largest size category. While for most techniques this increase is small, for EXP and CON the largest size category shows Median APEs that are among the highest of any size category, especially for 20 year projection horizons.

What accounts for this decrease in accuracy for the group of counties with the largest populations? There is no conceptual justification that could explain it. Tayman et al. (1998:9) note that the relationship between the size of the base population and forecast precision is a consistent finding in the forecast literature – the larger the base the smaller the error, and vice versa – and that instances of past studies that found deviations to this pattern at the very large or very small size categories were due to their lack of sufficient cases in those categories. In order to check whether the increase in Median APE shown in Figure 1 results from small sample size, the APEs for individual counties within each of the two largest size categories were plotted separately for each target year for 10 and 30 year projection horizons for the EXP method (data not shown). Both groups had a similar shape in the distribution of the individual county APEs. While for most target years the two size categories showed comparable Median APEs, in several years the accuracy of the projections for the largest size category was clearly lower than that for counties in the 30,000 to 100,000 size group. But none of the individual target year charts pointed to small sample size as the root cause of this unexpected finding.

The real explanation for the elevated Median APEs in the largest size category lies with the confounding effects of the underlying growth rates of counties in each size category. When the Median APEs for the two largest size categories were disaggregated by the population growth rates observed over the base period, within each growth category the results were comparable between the two size groups. However, the proportion of counties of the two largest size categories that fell into the highest growth category varied significantly. For every target year there were proportionately more counties in the >100,000 size category that experienced growth rates exceeding 25% (22.5% to 55.9%) than was the case for the 30,000 to 100,000 group (10.4% to 21.0%, respectively). As will be shown in more detail below, the highest growth rates are generally associated with the largest forecast errors, especially for the exponential method. The higher Median APEs for the largest size category shown in Figure 1 are therefore due to the fact that these counties experienced higher growth rates, on average, than counties in the next smaller size category over the base period, and not to population size effects as such.

Figure 2 is structured analogously to Figure 1, but focuses on population growth rates observed during the base period. The figure differentiates between ten population growth regimes: four that involve population losses (< -15%, -15% to -10%, -10% to -5%, -5% to 0%), and six that represent population gains (0% to 5%, 5% to 10%, 10% to 15%, 15% to 25%, 25% to 50%, >50%). The chart confirms the well-known u-shaped form of the relationship between forecast accuracy and population growth: errors are largest for counties at both ends of the growth spectrum – those that experienced significant declines and those that grew rapidly. The individual projection techniques differ with respect to whether high rates of population decline or growth produce larger errors in forecast accuracy. SFT, and to a lesser extent LIN, show higher errors for counties with strong population declines, while for EXP the fastest growing counties are projected with the lowest accuracy. These relationships also hold for longer projection horizons. However, it needs to be pointed out that the growth categories are not symmetrical. There are more categories of counties that experienced population growth (6) than those that declined (4). Furthermore, the top growth category (>50%) is significantly greater numerically than the top category representing population declines (<-15%).[8]

In a separate analysis (data not shown) the projections were rerun with symmetrical growth categories (< -15%, -15% to -10%, -10% to -5%, -5% to 0%, 0% to 5%, 5% to 10%, 10% to 15%, >15%). For 10 and 20 year projection horizons, the Median APEs for EXP and CON were very similar for each corresponding growth category (e.g. -10% to -15% vs. 10% to 15%), but for longer projection horizons for EXP the largest growth category (>15%) produced significantly greater errors than its corresponding category on the negative end. For LIN and especially SFT, negative growth rates were generally associated with larger errors. For SFT, the differences were very pronounced in that even the largest growth category (>15%) produced smaller errors than all negative growth categories except -5% to 0% for all projection horizons. This reiterates the caution that is advised in using ratio methods when the population change patterns in the smaller area are in the opposite direction of those of the larger area (for a discussion of this issue see Smith et al. 2001:179–180). In this study, the larger area was always the nation, which experienced population growth in each base period. Thus, the sign issue with the ratio techniques only applies to negative county growth rates. In these instances, higher growth in the nation would lead to larger declines in the county, which probably is not very realistic. Although there are ways to adjust the ratio techniques in these cases, none are completely satisfying, and the high errors associated with the SFT technique should be interpreted in this light.

[8] The categories were chosen to provide the most amount of detail given the limitations imposed in order to have a sufficiently large sample for the calculation of the error measures. Showing forecast error for counties with high rates of population growth was considered more important than achieving perfect symmetry between the categories on both sides of the growth spectrum.

For counties with growth patterns close to zero, most methods provide comparable results for shorter projection horizons, though SFT showing larger errors on the negative side. For rapidly declining counties, EXP and CON, as well as AV7 provide projections with the lowest errors in terms of accuracy. In fact, for counties that declined more than 15%, CON provides the lowest Median APE of any technique at every projection horizon, and the technique is very competitive for counties in the other categories with population declines as well. CON also does well for high growth counties, especially for longer projection horizons. The latter seems counterintuitive, but may be explained by the commonly observed tendency of extreme growth patterns to moderate over time, showing a regression toward the mean (Smith et al. 2001:319–320). Thus, while CON generally under-projects the population of those counties (see below), many of the other methods produce projections that are too high, which results in elevated APEs. This is particularly true for EXP, the technique that generates by far the greatest forecast errors for fast growing counties. Finally, Figure 2 also demonstrates the value of averaging. For every projection horizon and every growth category, AV7 shows Median APEs that compare very favorably to those of the primary projection techniques.

The discussion above regarding the uncharacteristically high Median APEs registered in the largest size category stresses the importance of accounting for both size and growth in an analysis of forecast error. When both characteristics are accounted for simultaneously, the above described relationships to forecast accuracy stay the same. Within each size category, there remains a u-shaped relationship depending on the growth pattern over the base period, and smaller counties tend to have greater forecast errors than larger counties irrespective of population growth (data not shown).

Figures 3 and 4 are analogous to Figures 1 and 2, but this time focusing on the bias of the projections as represented by the Median ALPE. Figure 3 shows that for LIN and SFT there exists a consistent relationship between population size and projection bias. While all size categories but the largest were under-projected, bias was greatest for the smallest counties. EXP and AV7 follow a similar general pattern, but the relationship is not as clear-cut. CON stands out in showing an essentially inverse relationship compared to the other techniques. For most techniques, extending the projection horizon from 10 to 20 years accentuates the ten-year pattern.

Figure 4 displays the Median ALPE by growth rate. Similar to the analysis by population size, the relationship between the county characteristic and bias varies in a consistent fashion, but the relationship appears to be even stronger and follows a clear stepwise pattern in which the Median ALPE either continuously increases or decreases along the growth spectrum. Once more, LIN, SFT, and EXP follow a similar pattern with CON going in the opposite direction. For counties that experienced population declines over the base period, LIN, SFT, and EXP have a tendency to under-project the target population while counties that grew are likely to have projections that turn out too high with these methods. This result lends further support to the notion that population change patterns moderate over time, i.e. regress towards the mean (Smith 1987). SFT shows the most negative bias for declining counties while EXP produces the most positive bias on the other end of the growth spectrum. Clearly, EXP and SFT should be used with caution for counties with these growth patterns. LIN, while slightly more biased than EXP for the declining counties, shows the lowest bias overall of the primary projection techniques. AV7 once more performs well, exhibiting levels of bias as low as or lower than the primary projection techniques.

As with the analysis of forecast accuracy, accounting for population size and growth simultaneously produces the expected results with respect to bias: Within each size category, counties with declining populations over the base period tended to get under-projected with most methods (except for the constant techniques, which again exhibit an inverse relationship), while smaller counties exhibit more bias than larger counties, controlling for population growth (data not shown).

There are several ways to look at bias. In addition to the algebraic percent error, the percent of all projection errors that are positive (or negative) is sometimes used to determine the tendency of a technique to either under- or over-project. Whereas the Median ALPE considers the magnitude of bias, the percent positive measure focuses on the overall tendency of a projection to be too high or too low. It is briefly mentioned here because it illuminates a different angle of bias. In a companion study by the author it was found that for most methods, the two error measures provide comparable results (Rayer 2005). For CON, however, the percent positive measure leads to a different conclusion. As described above, somewhat surprisingly, CON performed quite well both with respect to accuracy and bias. Although the method consistently under-projects counties that grew over the base period, CON is less plagued than most other techniques to take on large values of error, which manifest themselves especially for longer projection horizons. However, when looked at from the perspective of the proportion of all county projections that are too high or too low, the method showed more bias than most. Thus, while CON avoids large errors, the method is quite biased in that it under-projects a significant proportion of all counties with particular growth regimes. This serves as a reminder that one should look at more than one summary measure of error before making a determination regarding the choice of the appropriate projection technique for the task at hand.

CONCLUSION

This study of county population projections pursued two primary objectives. It first examined the relationship between the length of the base period and the length of the projection horizon with respect to forecast accuracy and bias. The main finding was that the length of the base period has a rather limited impact on the accuracy of the projections. The results do not support the notion that the length of the base period should correspond to the length of the forecast horizon, as Alho and Spencer (1997) suggest. Nor are 10 years necessarily sufficient, as Smith and Sincich (1990) recommend, because extending the base period to 20 years yielded some improvement in accuracy for most methods, and not just for EXP and SFT. Further lengthening had a negligible effect. For the exponential method, which can have a tendency to produce extremely high population forecasts, especially for the longer projection horizons, lengthening the base period had a considerable impact on individual county projections, albeit this was almost impossible to discern when using a robust measures of forecast error.

The observed relationship between the length of the base period and bias was deemed to be spurious. Bias is highly dependent on the choice of the launch and target years, and the discrepancy between population change patterns over the base period and those over the projection horizon. This confirms the conclusions of Smith and Sincich (1991), who found no consistent errors with respect to bias.

The second objective of the analysis was to assess forecast errors of the various trend extrapolation techniques. First, the methods were examined for the entire set of counties. This was followed by a more detailed analysis where counties were grouped by population characteristic, i.e. population size at the launch year, and the rate of population growth or decline over the base period. Building upon the findings regarding the impact of base period length on forecast accuracy and bias, the analysis was restricted to twenty year base periods. To make the discussion of results more manageable, the final part of the paper was further limited to a subset of the ten individual projection techniques, and the primary focus was on the ten and twenty year projection horizons.

The aggregate analysis revealed that many of the projection techniques produced comparable results, though SFT and COS showed significantly higher Median APEs. All three averages were highly competitive with respect to forecast error, being as good as or better than any individual method. There was slightly more divergence among the methods in terms of bias. As was true for accuracy, SFT and COS produced the greatest errors: SFT greatly under-projected the population and projections made with COS came out much too high overall. CON had a distinct negative bias, which makes sense given that most counties have grown over the past century. EXP showed the least bias, but that only applied when a robust summary measure of error was used.

The disaggregate analysis of forecast accuracy and bias by population size and growth confirmed many of the findings of earlier studies, but did so using a significantly enlarged data set with respect to time and space, thus strengthening their conclusions. The analysis showed that both size and growth impact the projections. As expected, with increasing population size the projections become more accurate. However, most of the improvement comes at fairly low levels of population size. The accuracy of the projections actually decreased for counties in the largest size category, but this result was shown to be driven by this county group's underlying growth characteristics. Indeed, it could be argued that except for perhaps the smallest counties population size alone is only a limited indicator of forecast accuracy, and that growth dynamics are of greater importance. With respect to the latter, the study revealed an essentially perfectly u-shaped relationship for all methods, with forecast accuracy being lowest for counties that were declining or growing rapidly and highest for counties with little change in either direction over the base period. Individual techniques varied in that some produced greater errors for counties experiencing population declines while others were significantly less accurate for fast growing counties. As in the aggregate analysis, forecast accuracy decreased with increasing projection horizons. The joint analysis by both size and growth confirmed these relationships in general.

In addition to variations in forecast accuracy, the individual projection techniques also revealed differences in bias by size and growth category. Once again, the relationships were stronger with respect to population growth than with size. Perhaps the most striking finding of the entire paper was the consistent stepwise pattern between bias and population growth shown in Figure 4. For counties that experienced population declines over the base period, LIN, SFT, and EXP all under-projected the target year populations, while those that grew were projected too high. The stronger the population declined or increased over the base period, the more negative or positive the bias of the ensuing projections. This result underscores the tendency of population growth patterns to moderate over time and lends strong support to the notion of a regression towards the mean. The analysis was also instructive in showing that one group of techniques, i.e. those that hold the population constant, follow a different path. While not necessarily suitable as a primary choice of projection method, when used in combination with the other methods – as was done in this study in the form of three averages – the outcome can be positive.

The generally good performance of the three averages was another major finding of this study. Throughout, the methods that involved the averaging of all the primary projection techniques, or a trimmed version thereof, showed among the highest levels of accuracy and the lowest amount of bias of any technique. Averaging techniques have long been advocated for forecasts in various fields (see e.g. Armstrong 2001; Makridakis et al. 1982; Webby and O'Conner 1996), but have been surprisingly rare in population projections. The results from this study, executed for a large sample of counties and for a very long time horizon, should encourage practitioners of population projections to more seriously consider combining different projection techniques.

While all three averages provided good results, somewhat surprisingly AV7, which includes all seven primary projection techniques, in many instances came out on top. The reason for this has to do with that particular technique incorporating a variety of methods that each have their own strengths and weaknesses, and which counterbalance each other. This was most apparent in the discussion of bias where the constant techniques showed the opposite pattern of the remaining primary techniques with respect to size and especially growth. In this sense, using both types of methods in a projection – those that involve some aspect of constancy and those that reflect change – works in a similar fashion to a diversified investment portfolio that includes stocks and bonds or other types of securities the returns of which are negatively correlated. The trimmed averages often exhibited more bias and lower levels of accuracy because they excluded exactly those beneficial "outliers." In practice, though, one has to weigh the benefits of this finding, especially when looking at individual projections, for it may well turn out that the low Median APEs reported for AV7 mask a number of unreasonable county projections – particularly those produced by SFT and EXP at both ends of the growth spectrum – and that a mean-based measure of error might lead to a different choice of projection technique. Yet none of this should detract from the potential of averaging, which for many purposes can lead to more satisfactory projections than any particular primary projection technique by itself.

Another avenue worth further investigation involves, rather than indiscriminately calculating an average out of all techniques, or a trimmed version thereof, the development of a targeted average based on those methods shown to be the most appropriate for a county's particular size and growth profile. This is analogous to the "composite" approach as advocated by Isserman (1977). Smith and Shahidullah (1995) applied this process for census tracts and found that excluding EXP for fast growing places and SFT for slowly growing and declines places produced more accurate projections than a simple average, but comparatively little research has been devoted to this issue. The results from the present study suggest that counties that experienced population declines over the base period might best be served by the exponential and/or constant method. At the same time, the exponential method had the greatest difficulty with fast growing counties, and should probably not be used for these counties. This analysis took first steps towards the development of a more targeted approach to choosing projection techniques appropriate for the underlying demographic processes of the study areas. Population projections inevitably involve many unknowns. We will never be able to perfectly project a population all of the time. However, the detailed analysis presented in this paper regarding the relationship between forecast accuracy and bias and selected county population characteristics revealed sufficiently consistent patterns that make the endeavor worth the try, and it is towards this goal that further research will be directed.

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Table 1. Impact of Starting the Analysis in 1900 vs. 1930: Comparison of Median APEs by Horizon and Method

N	Horizon	Dataset	LIN	MLN	SHR	SFT	EXP	COS	CON	AV7	AV5	AV3
20	10	1900-	7.9	7.8	8.2	10.6	7.9	11.1	9.5	7.3	7.5	7.7
20	10	1930-	8.3	8.1	8.6	11.2	8.2	11.5	9.8	7.6	7.8	8.1
15	20	1900-	14.9	14.4	16.0	23.1	15.0	21.0	17.2	13.5	13.8	14.5
15	20	1930-	15.5	15.0	16.7	24.2	15.6	21.5	17.7	14.1	14.4	15.1
10	30	1900-	24.9	23.6	27.1	41.8	24.1	31.0	24.0	21.9	22.7	23.9
10	30	1930-	26.1	24.6	28.4	43.6	25.0	31.4	24.6	22.7	23.6	24.8
6	40	1900-	31.9	30.2	35.3	56.5	30.7	41.3	29.6	27.1	29.2	30.4
6	40	1930-	33.3	31.5	36.9	59.0	32.0	42.1	30.2	28.3	30.2	31.8
3	50	1900-	35.4	34.2	38.4	55.7	34.0	45.4	35.1	30.1	32.8	34.3
3	50	1930-	37.2	36.2	40.3	58.6	35.6	46.6	35.7	31.1	34.0	36.0
54	All	1900-	17.2	16.5	18.6	27.4	16.8	22.8	19.9	15.2	15.9	16.6
54	All	1930-	18.0	17.2	19.4	28.7	17.5	23.3	20.4	15.8	16.5	17.3

Table 2. Median APE by Projection Horizon, Base Period, and Projection Technique

N	Horizon	Base	LIN	MLN	SHR	SFT	EXP	COS	CON	AV7	AV5	AV3
9	10	10	8.6	8.5	8.9	10.4	8.8	12.6	8.9	7.8	8.2	8.5
8	10	20	7.7	7.7	8.0	9.8	7.8	12.1	8.9	7.4	7.5	7.6
7	10	30	7.8	7.7	8.1	10.6	8.0	11.7	8.9	7.5	7.5	7.7
6	10	40	8.2	8.1	8.5	11.6	8.2	12.1	8.9	7.9	7.9	8.1
5	10	50	8.4	8.3	8.8	13.0	8.5	11.4	8.9	8.1	8.1	8.3
8	20	10	15.7	15.6	16.7	22.0	16.4	24.5	15.8	14.1	14.7	15.5
7	20	20	14.6	14.4	15.7	22.0	15.1	23.6	15.8	13.7	14.0	14.4
6	20	30	15.3	14.9	16.3	23.9	15.4	23.4	15.8	14.5	14.6	15.0
5	20	40	15.8	15.6	16.9	25.6	15.9	22.8	15.8	15.3	15.2	15.5
4	20	50	15.4	14.8	16.5	27.0	15.4	20.5	15.8	14.7	14.5	14.9
7	30	10	24.7	24.2	26.9	39.1	25.3	37.4	22.4	22.0	22.9	24.1
6	30	20	23.5	23.0	25.7	39.2	23.9	36.2	22.4	21.9	22.2	23.0
5	30	30	24.4	23.6	26.3	41.1	24.3	34.6	22.4	22.7	23.0	23.7
4	30	40	23.9	23.3	25.8	42.1	23.8	32.3	22.4	22.6	22.8	23.2
3	30	50	23.6	22.4	25.8	46.0	23.6	28.9	22.4	22.5	22.4	22.9
6	40	10	33.3	32.8	37.2	59.1	34.1	51.5	28.7	29.6	31.4	32.5
5	40	20	31.0	30.2	34.4	55.1	31.3	47.7	28.7	28.5	29.3	30.1
4	40	30	30.4	29.9	33.3	54.5	30.8	44.1	28.7	28.3	29.1	29.7
3	40	40	30.0	29.0	33.2	57.2	30.7	40.8	28.7	28.5	28.7	29.1
2	40	50	30.5	29.1	33.7	63.0	31.6	39.5	28.7	28.7	29.0	29.4
5	50	10	40.7	40.0	46.0	74.6	42.3	61.1	34.1	36.2	38.4	39.8
4	50	20	36.4	35.9	40.7	66.9	37.2	55.9	34.1	33.7	34.7	35.5
3	50	30	36.3	36.1	40.7	71.2	37.7	53.0	34.1	34.0	35.1	35.9
2	50	40	36.0	36.0	40.8	70.6	38.3	51.6	34.1	34.8	35.2	35.8
1	50	50	36.8	35.6	42.2	76.6	39.3	48.2	34.1	35.1	35.4	35.9

Table 3. Median ALPE by Projection Horizon, Base Period, and Projection Technique

N	Horizon	Base	LIN	MLN	SHR	SFT	EXP	COS	CON	AV7	AV5	AV3
9	10	10	-1.4	-0.7	-1.3	-3.3	-0.1	8.6	-3.4	-0.1	-0.7	-1.0
8	10	20	-1.7	-0.7	-1.6	-4.4	0.1	8.2	-3.4	-0.4	-0.8	-1.1
7	10	30	-2.1	-0.9	-2.0	-6.0	-0.2	7.8	-3.4	-0.9	-1.2	-1.5
6	10	40	-2.5	-1.0	-2.4	-7.6	-0.2	7.6	-3.4	-1.3	-1.4	-1.8
5	10	50	-3.4	-1.7	-3.3	-9.5	-0.9	6.6	-3.4	-2.3	-2.2	-2.4
8	20	10	-3.0	-1.0	-2.6	-8.8	0.8	18.5	-6.4	0.0	-1.2	-1.8
7	20	20	-4.3	-1.9	-4.1	-12.4	-0.4	17.5	-6.4	-1.5	-2.4	-3.0
6	20	30	-5.2	-2.2	-4.8	-15.4	-0.4	16.8	-6.4	-2.4	-2.9	-3.5
5	20	40	-5.9	-2.6	-5.5	-17.8	-0.5	15.2	-6.4	-3.5	-3.5	-4.1
4	20	50	-7.6	-4.0	-7.3	-20.9	-2.0	12.6	-6.4	-5.8	-5.3	-5.7
7	30	10	-7.9	-4.1	-7.3	-21.2	-1.7	29.3	-9.6	-2.9	-4.9	-5.7
6	30	20	-8.7	-4.4	-8.3	-25.2	-1.8	27.5	-9.6	-4.0	-5.3	-6.3
5	30	30	-9.9	-5.1	-9.4	-27.9	-1.8	24.5	-9.6	-5.5	-6.2	-7.1
4	30	40	-10.8	-5.8	-10.6	-30.7	-2.7	20.7	-9.6	-7.2	-7.2	-8.0
3	30	50	-14.0	-8.2	-13.6	-37.3	-6.0	16.6	-9.6	-10.8	-10.5	-11.0
6	40	10	-10.4	-4.7	-9.5	-32.8	-0.9	42.2	-11.9	-3.0	-6.2	-7.2
5	40	20	-10.4	-4.2	-9.6	-34.9	-0.1	36.8	-11.9	-3.5	-6.0	-7.0
4	40	30	-11.5	-4.9	-10.5	-36.1	-0.4	31.7	-11.9	-5.6	-6.9	-7.8
3	40	40	-13.1	-5.9	-12.7	-41.9	-2.3	26.8	-11.9	-7.9	-8.6	-9.4
2	40	50	-15.0	-7.2	-13.7	-48.8	-4.0	25.4	-11.9	-10.0	-10.4	-10.6
5	50	10	-11.5	-4.5	-9.9	-38.0	1.9	50.3	-14.6	-1.8	-6.3	-7.5
4	50	20	-11.1	-3.7	-9.6	-39.1	1.9	43.1	-14.6	-2.9	-5.9	-6.9
3	50	30	-12.7	-4.2	-11.2	-47.0	1.0	39.1	-14.6	-5.2	-6.8	-7.7
2	50	40	-12.9	-3.5	-11.3	-48.3	1.4	36.5	-14.6	-5.2	-6.9	-7.6
1	50	50	-16.5	-6.2	-14.6	-55.2	-1.8	32.2	-14.6	-9.4	-10.4	-11.1

Table 4. % Change in Population per Decade by Time Period

Time Period	Median All Counties	Mean All Counties	National Total
1900-1910	8.8	44.0	21.0
1910-1920	3.6	9.0	15.0
1920-1930	2.4	19.9	16.2
1930-1940	4.4	6.2	7.3
1940-1950	0.0	4.3	14.5
1950-1960	-0.2	6.0	18.5
1960-1970	1.9	5.2	13.4
1970-1980	12.2	16.3	11.4
1980-1990	1.3	4.0	9.8
1990-2000	8.4	11.1	13.1
1900-1920	6.3	17.2	17.9
1910-1930	3.3	9.1	15.6
1920-1940	3.3	8.2	11.7
1930-1950	2.3	4.6	10.8
1940-1960	-0.1	4.6	16.5
1950-1970	1.0	5.1	15.9
1960-1980	7.2	10.2	12.4
1970-1990	6.6	9.8	10.6
1980-2000	4.8	7.3	11.5
1900-1930	4.9	14.5	17.4
1910-1940	3.3	7.2	12.7
1920-1950	2.0	6.2	12.6
1930-1960	1.6	4.6	13.3
1940-1970	0.5	4.4	15.4
1950-1980	4.9	8.2	14.4
1960-1990	5.4	7.9	11.5
1970-2000	7.2	10.1	11.4
1900-1940	4.7	11.2	14.8
1910-1950	2.4	6.0	13.2
1920-1960	1.5	5.6	14.0
1930-1970	1.6	4.5	13.3
1940-1980	3.7	6.9	14.4
1950-1990	4.0	7.0	13.2
1960-2000	6.2	8.6	11.9
1900-1950	3.8	9.3	14.7
1910-1960	2.0	5.5	14.2
1920-1970	1.9	5.2	13.9
1930-1980	4.0	6.5	13.0
1940-1990	3.1	6.1	13.5
1950-2000	5.0	7.6	13.2

Table 5a. Median APE by Projection Horizon and Projection Technique, 20 Year Base Period

N	Horizon	LIN	MLN	SHR	SFT	EXP	COS	CON	AV7	AV5	AV3
8	10	7.7	7.7	8.0	9.8	7.8	12.1	8.9	7.4	7.5	7.6
7	20	14.6	14.4	15.7	22.0	15.1	23.6	15.8	13.7	14.0	14.4
6	30	23.5	23.0	25.7	39.2	23.9	36.2	22.4	21.9	22.2	23.0
5	40	31.0	30.2	34.4	55.1	31.3	47.7	28.7	28.5	29.3	30.1
4	50	36.4	35.9	40.7	66.9	37.2	55.9	34.1	33.7	34.7	35.5
30	All	20.2	19.8	22.1	33.7	20.6	31.4	20.3	18.8	19.2	19.7

Table 5b. Median ALPE by Projection Horizon and Projection Technique, 20 Year Base Period

N	Horizon	LIN	MLN	SHR	SFT	EXP	COS	CON	AV7	AV5	AV3
8	10	-1.7	-0.7	-1.6	-4.4	0.1	8.2	-3.4	-0.4	-0.8	-1.1
7	20	-4.3	-1.9	-4.1	-12.4	-0.4	17.5	-6.4	-1.5	-2.4	-3.0
6	30	-8.7	-4.4	-8.3	-25.2	-1.8	27.5	-9.6	-4.0	-5.3	-6.3
5	40	-10.4	-4.2	-9.6	-34.9	-0.1	36.8	-11.9	-3.5	-6.0	-7.0
4	50	-11.1	-3.7	-9.6	-39.1	1.9	43.1	-14.6	-2.9	-5.9	-6.9
30	All	-6.4	-2.7	-5.9	-20.1	-0.2	23.6	-8.6	-2.2	-3.6	-4.3

Figure 1. Median APE by Population Size, 20 Year Base Period, 10 & 20 Year Projection Horizon (<2.5, 2.5-7.5, 7.5-15, 15-30, 30-100, >100k)

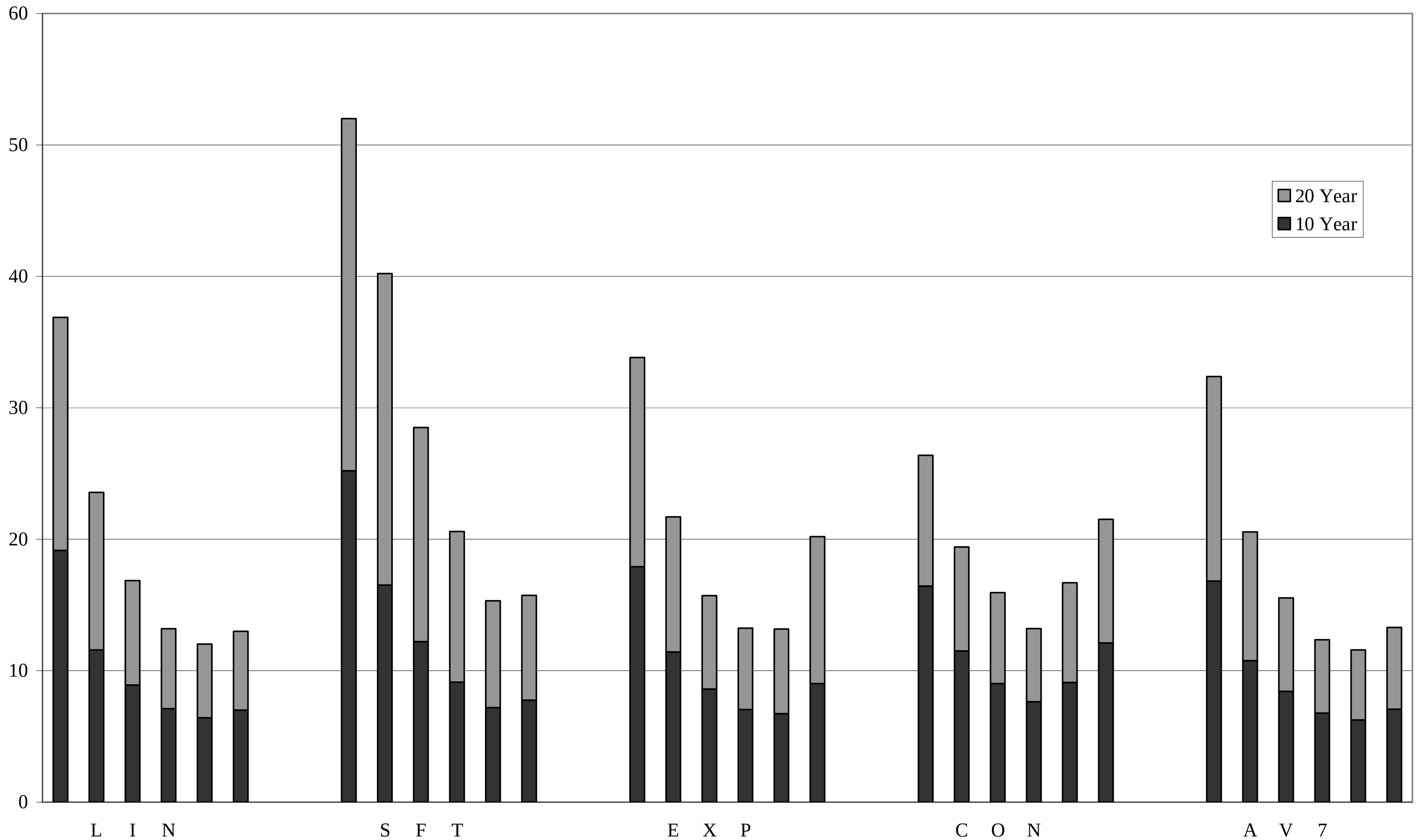

Figure 2. Median APE by Population Growth, 20 Year Base Period, 10 & 20 Year Projection Horizon (<-15, -15 to -10, -10 to -5, -5 to 0, 0 to 5, 5 to 10, 10 to 15, 15 to 25, 25 to 50, >50%)

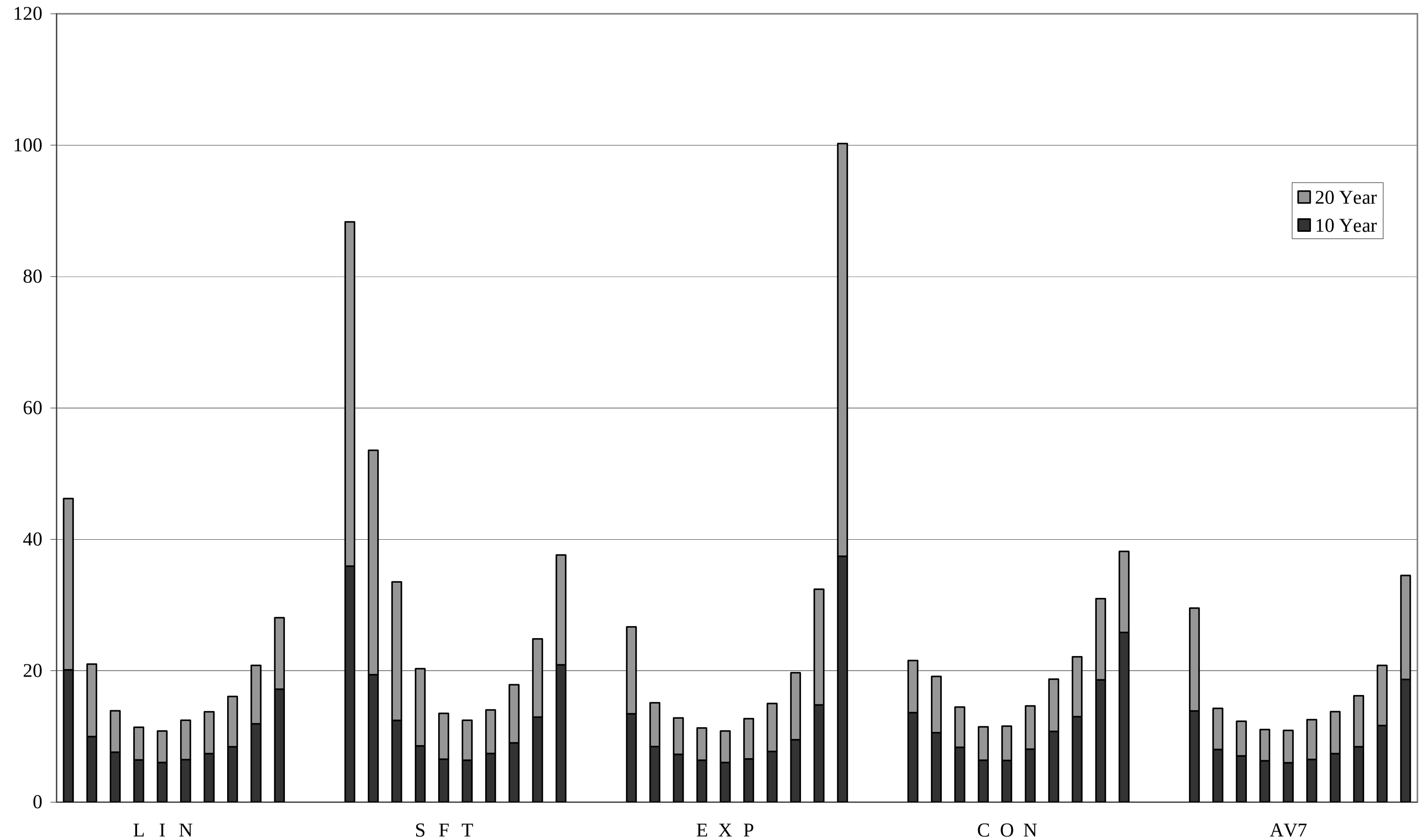

Figure 3. Median ALPE by Population Size, 20 Year Base Period, 10 & 20 Year Projection Horizon (<2.5, 2.5-7.5, 7.5-15, 15-30, 30-100, >100k)

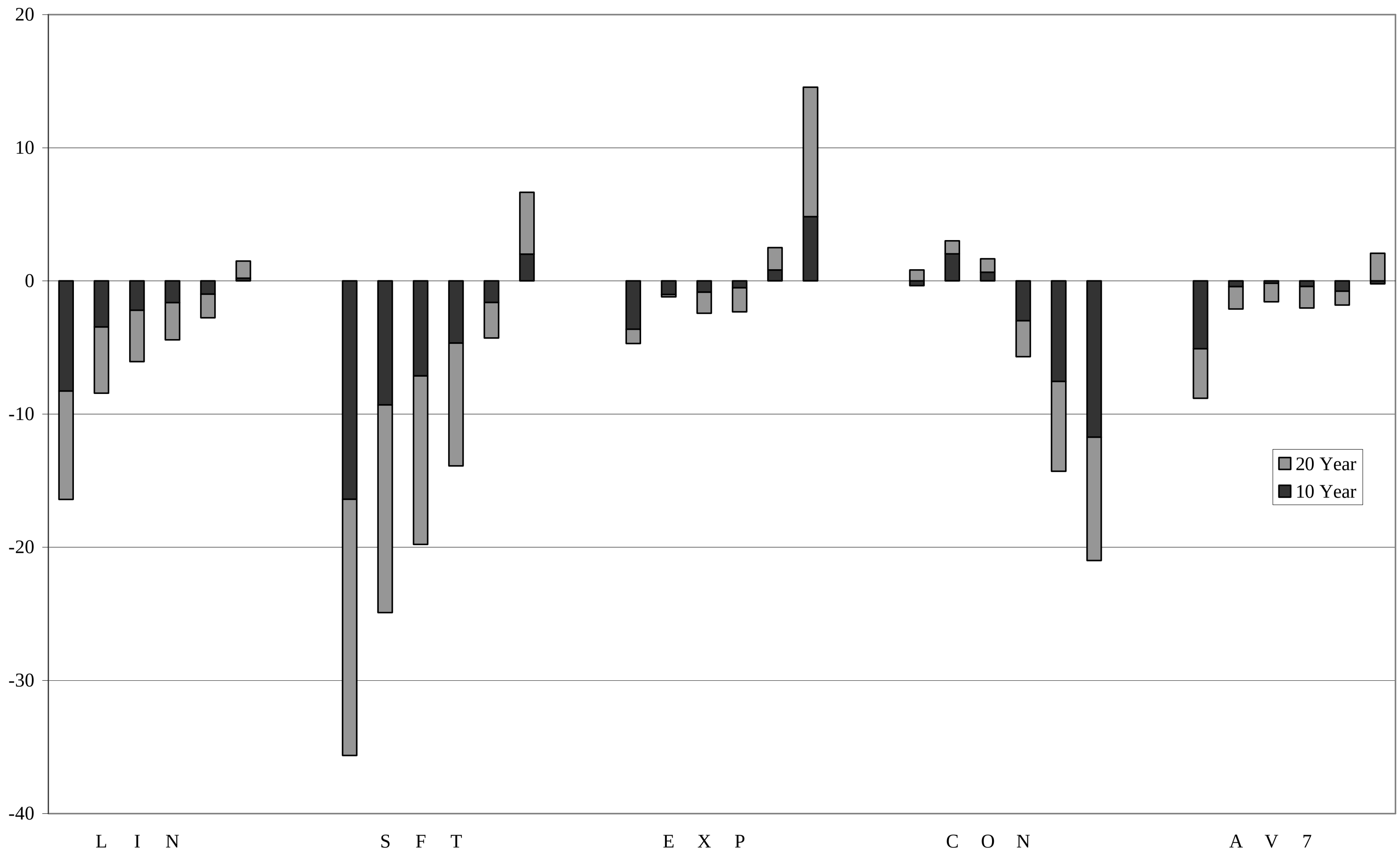

Figure 4. Median ALPE by Population Growth, 20 Year Base Period, 10 & 20 Year Projection Horizon (<-15, -15 to -10, -10 to -5, -5 to 0, 0 to 5, 5 to 10, 10 to 15, 15 to 25, 25 to 50, >50%)